## VARIATIONS on GOLF CROQUET



# VARIATIONS on GOLF CROQUET 

written by ${ }^{1}$

## Howard Sosin

with

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Acknowledgements ${ }^{2}$

To Carmen and Matthew. Here's hoping that the actual existence of this book explains some of my absence. Howard

To Sherif, who introduced me to the wonders of Golf Croquet. And to my family, for their patience and for cheering me on. Ben

[^0]
## FOREWARD by Stephen Mulliner

Golf Croquet (GC) is growing in popularity and has become the life-blood of many croquet clubs around the world. This is because GC is approachable and social, demanding and exciting, and just plain fun at all levels of play. It ain't broken, so why introduce variations? The goal is to create new strategic and shot-making challenges that complement and augment traditional ones. This book presents four new games that do just that:

2-Shot: As in GC, the $1^{\text {st }}$ shot of a turn can be used for any purpose. In addition, if the $1^{\text {st }}$ shot involves a roquet of any ball, then a $2^{\text {nd }}$ shot is earned and can once again be used for any purpose. Game is played to 5 points. The intrigue of 2-Shot is the $2^{\text {nd }}$ shot. Once earned, it adds new dimensions to play. Chief among these is the ability to use another ball as a "pioneer" to aid in offense or to use it as a "Stepping-Stone" to aid in defense.

Dueling Duos (DD): The order of hoops is determined by random draw - there is no offsides. Play at a hoop continues until one team scores it with both balls. Teams earn one point for each ball that scores and an additional point ( 3 in total) if both of their balls score while opponents score none. Game is played to 8 points. Balls that score continue to play in rotation but with changed priorities, going from offense to defense. Scoring twice before your opponent scores once is extremely valuable.

3-Shot: Each turn involves two shots and a possible $3^{\text {rd }}$ which is earned if there is a roquet on the $1^{\text {st }}$ or $2^{\text {nd }}$ shots. A hoop can be made only after a roquet. One point is earned at each hoop. Game is played to 5 points. Scoring often begins with a long roquet with the $1^{\text {st }}$ shot that is followed by two others: a lag to the Current-Hoop, and a hoop shot. Failure to have an initial roquet usually leads to defensive play that seeks to isolate the Danger-Ball.

AC-GC: A team can play either ball each turn, but in order to score it must play both balls in the Period of the Current-Hoop. The team losing at a hoop always plays first to the next one. Game is to at least 5 points. Winning requires scoring one more time when Oppos goes first than Oppos score when you go first. This is difficult and often requires cunning and "gambling" to succeed.

I believe these new games will increase overall interest and play, thereby promoting and helping to secure the future of all croquet.

## FOREWARD by Ben Rothman

I am often asked which version of Croquet I like best. While I play Golf Croquet (GC), Association Croquet (AC), and American Rules (US) tournaments, for normal play or practice with friends I opt for GC. Practicing AC or US is a solitary pursuit and even if I desire a game, the long turns do not offer much for the out-player. GC has very little time between shots causing all players to stay near the lawn and chat making it social and interactive. These qualities persist in the variations offered in this book.

Each new game rewards different skills and introduces tactics that are drawn from AC and US: 2-Shot encourages players to control their pace and the exact angle of a roquet; Dueling Duos minimizes the chances to "steal" a hoop as players must control the area long enough to score both balls and introduces the concept of a ball becoming dedicated to defense after it scores; 3-Shot often requires a long hit-in followed by a careful lag to distant hoops and then a choice of attempting the hoop or retreating to partner; and AC-GC allows players to play their best ball on each turn while introducing strategies that can be considered "gambling" and an innovative "Both Balls Rule" to avoid degenerative play.

These variations create new and intriguing challenges without requiring players to perfect 2-ball croquet shots. They will sharpen your GC play and should become favored games in and of themselves whether the goal is for social or competitive play.

As always, have fun and play well!

## FOREWARD by the Fearsome Foursome

Howard has invented four GC-based games. He introduced 2-Shot in March 2019 during the 2 ${ }^{\text {nd }}$ Croquet Innovations Tournament (CIT II) and had planned to introduce the other three (Dueling Duos, 3-Shot and AC-GC) in March 2020 at the CIT III but it was cancelled because of Covid-19. Instead, they were introduced in smaller, socially-distanced, all-local events, that took place in November and December 2020 at the NCC in Palm Beach. In our sessions we (the "Fearsome Foursome"):

1. Took skill tests that Howard devised. Our results are shown and were used to develop probabilities in support of alternative strategies described in this book.
2. Played round-robin doubles matches, one for each of the new games. These were captured on videos that can be seen on the YouTube Channel, Croquet Innovations.

Quite frankly, we were skeptical - what could possibly be done to augment/advance/diversify GC? As we saw the rules we wondered: How much could the possibility of a second shot during a turn really matter? Besides slowing play, what possible intrigue could arise by requiring that both balls score at a hoop before proceeding to the next one? Once you added a second shot, wouldn't it be gilding the lily to add a third? And after all of that, isn't it almost sacrilege and needlessly confusing to allow either ball of a team to play each turn?

In the end, we were surprised and delighted by how seemingly simple yet subtle changes to the rules of GC created games with amazing brain-teasing diversity - games in which we were all challenged once again to think (and argue!) and perfect new skills.

We recommend these games to you. Enjoy!

Sherif Abdelwahab<br>Matthew Essick<br>Danny Huneycutt<br>Stephen Morgan

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## INTRODUCTION ${ }^{3}$

This book was written for Croquet Players who are already play and enjoy Golf Croquet (GC). It describes four new games. These involve simple changes to the rules of GC that yield new and exciting challenges in the context of the familiar.

There are five chapters and two appendices:
Chapter 1 is entitled "Making and Implementing Strategic Decisions". We use Ben's "SHIP List" as our decision making tool where SHIP is an acronym for: Shoot the Hoop, Hit a Ball, Interfere with a Ball, and Take Position. It is an easy to remember and intuitive tool that Ben teaches to beginners and advanced players alike and uses in his own play. We show how it works for GC and how it can be adapted to decision making in each of the new games.

The execution of any shot ultimately relies on "feel". We describe two new inputs that help find this "magic": (i) Knowledge of standard distances on a croquet court, and (ii) The results of simple skills tests that relate these distances to your Critical Distance.

Four chapters follow, one for each new game: Chapter 2: 2-Shot, Chapter 3: Dueling Duos, Chapter 4: 3-Shot, and Chapter 5: AC-GC. Each chapter provides rules, strategic notes, observations from videos, and SHIP analyses for the game at hand.

These chapters are followed by two Appendices: The $1^{\text {st }}$ is devoted to a tournament format that involves five games (GC and the four new ones). The games to be played in a match are not known ahead of time. Instead, they are determined by random draw during each match. The $2^{\text {nd }}$ describes a new handicap system, that we call the Quarter System. It works for traditional GC and these new games. It allows weaker players to "improve" their lagging and clearing results without gaming the system.

## Reading This Book

We begin with a chart that tabulates rule changes, and then provide a link to videos for all of the games. We suggest that you glance at the rule changes and pick a game that interests you. Then watch a video (or two!) and spend time on the lawn, hopefully with friends, "giving it a go". After that, work through the relevant chapter considering the rules and the strategic notes. This way of proceeding will expedite your progress toward understanding, mastering, and enjoying these games.

[^1]
## Rule Changes

Each Chapter begins by presenting the current rules of the game at hand. We say "current" because the rules have evolved, and with play may benefit from additional tweaks. Comments/suggestions are welcomed - howard@sosin.net.

| ITEM | GC | 2-SHOT | DD | 3-SHOT | AC-GC |
| :---: | :---: | :---: | :---: | :---: | :---: |
| First to Next | GC | GC | GC | GC | Loser |
| Hoop Order | GC | GC | Random | GC/random | GC/random |
| Offsides | GC | GC | None | None | None |
| Continuous Play | GC | AC | AC | AC | AC |
| Mark-in | GC | AC | AC | AC | AC |
| Striking Faults | GC | AC | AC | AC | AC |
| Start-in Area | GC | baulk-line | baulk-line | 4 corners | baulk-line |
| $1{ }^{\text {st }}$ Ball in Game | GC | Bamford | GC | GC | u |
| Winning Score | GC | 5 @ h(12) | 8 | 5 | 5+ |

First to Next: In GC, the team that loses at a hoop normally goes first to the next one. There is an exception - the winning team goes first if the hoop was won as the result of a peel by opponents. This rule and its exception are maintained in 2-Shot, DD, and 3-Shot, but the exception is intentionally removed in AC-GC.
$1^{\text {st }}$ Hoop and Hoop Order: In DD, all hoops are chosen and played in random order. AC-GC starts at h(1). 2-Shot and 3-Shot randomly choose the initial hoop. The later three games all progress in standard hoop order wrapping around from $h(12)$ to $h(1)$ if necessary.

Offsides: Offsides in GC are assessed after every score to prevent players from "going ahead" to the next hoop. This procedure is followed in 2-Shot, but not in the other games. It is not needed in 3-Shot. There are no offsides in DD because the next hoop is not known while the Current-Hoop is being played. Finally, offsides does not apply in AC-GC because the goal is to promote simultaneous play at multiple hoops.

Continuous Play: All four of these games adopt the AC rule: Once in the game, a ball is never an outside agency. Continuous play facilitates clearing shots on otherwise borderline balls.

Mark-in: These games follow AC rules for marking balls in 1-yard. This allows borderline balls to be roqueted/rushed.
Striking Faults: The games adopt AC rules for striking faults. This adds strategic options by making some double-hits legal.
Start-in Area: 2-Shot, DD, and AC-GC use a Start-in Baulk-Line. It runs for six yards along the east boundary starting 1-yard north of c4. The baulk line allows different angles of attack to the First Hoop. 3-Shot has the balls start from their colored-corners -u in c 1 , $r$ in $\mathrm{c} 2, \mathrm{k}$ in c 3 and y in c 4 .
$1^{\text {st }}$ Ball in Game: Going first in GC is an advantage that is bestowed on the team that wins the coin toss. This structure is maintained in DD and 3-Shot but modified in the other games. 2-Shot has a "Bamford-Start" wherein one team plays an initial ball and then the other team can switch colors and take over that ball as their own. AC-GC requires $u / k$ to play first. This allows a definition of the winning score that mitigates the right to go first.

Winning Score: Each game has its own scoring system ${ }^{4}$ :
(i) In 2-Shot is played to 5 points.
(ii) In $\underline{D D}$ teams earn one point for each ball that scores at a hoop and an additional point (three in total) if both of their balls score while opponents score none. The winner is the first team to reach 8 points.
(iii) 3-Shot is played to 5 points.
(iv) AC-GC is played to at least 5 points. $u / k$ plays first to start the game and must win by two while $r / y$ can win by one. This unusual scoring system requires the winning team to "break-serve" - win a point when opponents go first - at least one more time than the other team does. That this is "fair" is explained at length in the text.

[^2]
## Let's Go to the Video Tapes ${ }^{5}$

Videos with narrations are available for each of the games. They can be seen on the YouTube Channel "Croquet Innovations" ${ }^{6}$.
2-Shot: There are six videos to choose from. The first three are taken from CIT II played in March 2019 - two early matches, Jamie Burch vs Danny Johnston, and Ben Rothman vs Pete Trimmer, and then the finals, Jamie Burch vs Pete Trimmer. The last game is announced by Stephen Mulliner and crowns Pete as the reigning 2-Shot champion. The final three videos come from a match between Sherif Abdelwahab and Jeff Soo played in May 2019, with Sherif carrying the day.

DD, 3-Shot, and AC-GC: Initial videos for these games come from Round-Robin Doubles Matches involving the Fearsome Foursome that were held in November and December 2020. Here are the results:

|  | AC-GC |  |  | DD ${ }^{8}$ |  | 3-SHOT |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Gam | me | ame | Game 1 | Game 2 | Game 1 | Game | Game 3 |
| Sherif | w | -- | w | w | w | w | -- | -- |
| Matthew | -- | w | w | -- | -- | w | w | w |
| Danny | w | w | -- | -- | w | -- | -- | w |
| Stephen | -- | -- | -- | w | -- | -- | w | -- |

Additional videos for AC-GC and DD were added in October 2021 involving just half of the "Foursome", the "Twosome", Matthew and Stephen playing singles. These are discussed in the text.

[^3]
## Chapter 1: MAKING AND IMPLEMENTING DECISIONS

There are two key elements of success in all GC-based games - (i) analyzing situations out on the lawn and (ii) implementing your chosen strategy through shot selection and execution.

## Decision-Making with the SHIP List

Ben invented, uses, and teaches decision making in GC by proceeding through his "SHIP List" which is an acronym for: Shoot the Hoop, Hit a Ball, Interfere with a Ball, and Take Position. Here we present SHIP for GC. Then, at the end of the chapter for each game, we present appropriate modifications.

## The SHIP List for GC

1. Shoot the Hoop: If you have a hoop shot, consider taking it regardless of the other ball positions. This is the only way to score points and win the game. If the shot is unlikely to succeed and there are other balls in position to score, then:
2. Hit a Ball: Consider the order of play. If it has a hoop shot, then consider hitting the Danger-Ball ${ }^{9}$. The Danger-Ball may need to be hit as hard as possible, to limit its threat to Striker or Partner but sometimes it is only necessary to take away the Danger-Ball's hoop shot and hitting it harder does not offer any real tactical advantage (it can just come back). If you can improve the position of your Partner-Ball (and the Danger-Ball cannot move it), then consider promoting Partner. If the Spent-Ball is the only ball in good position, consider moving it. If none of these shots are likely to succeed, then:
3. Interfere with a Ball: Maybe you can block the Danger-Ball or go to where it is blocked from hitting you. Maybe you can go so close as to smother all of its useful shots. Look to your Partner-Ball and avoid interfering with shots it might prioritize. Maybe the Spent-Ball is the only thing you can interfere with. If so, do it. If not:
4. Take Position: Most of the time this is position at the hoop, and often there is value in taking a less than ideal position (to avoid giving the opponent a stop shot clearance). Occasionally this position is behind the hoop, towards the corner, or halfway to the next hoop when there are extenuating circumstances.
[^4]Any time Striker can satisfy multiple priorities in one shot, he can take control of the Current Hoop. This often happens with stop shots, where the Striker is able to clear Oppo and remain in scoring position. This can also be achieved with an effective block, where the Striker gets good position at the hoop and interferes with Oppo's hoop shot. These canny plays turn defense into offense and wrest control of the current hoop from the adversary.

Many advanced plays are the end result of satisfying multiple priorities simultaneously when already in control of the Current Hoop. Jawsing at an odd hoop is shooting the hoop and getting position. The in-off is hitting a ball and shooting the hoop. Promoting Partner to a wired position is hitting a ball and interfering with the Danger-Ball's shot. Clearing an opponent to a blocked position is also hitting a ball and interfering. Promoting Partner through the hoop while ricocheting to the next hoop is hitting a ball and getting position. Offensive multitasking helps win contentious and multiple hoops.

## Implementing Decisions: Inputs to the Development of Feel

Introduction: The execution of any shot ultimately relies on "feel". Each player has his own way of developing this magic. It can be by practice, discussion, visualization, play, videos, or some combination of these. Presented here are two additional and related inputs you can use: (i) knowledge of Standard Distances on a croquet court, and (ii) the results of simple skills tests that relate Standard Distances to your Critical Distance - the distance at which you are able, on average, to roquet a ball $50 \%$ of the time. Together these can be useful inputs to the development of feel and the selection of sound strategies, especially for those of us who are data and analysis driven.

## Standard Distances

Shot distances can be measured by walking them off (assuming the length of your stride is known and constant!) but doing this each turn is tedious. Instead, easily observed Standard Distances between known landmarks (corners and penalty spots) and croquet's furniture (the six hoops and the peg) can be internalized/memorized and added together quickly during play to provide very good estimates of non-standard distances. Our list of useful distances along with our most basic unit of measure - hash marks every 3.5 yards - are shown in the diagrams below. The distances are shown in yards, rounded to whole integers - the way we remember them - along with one decimal point detail (for the obsessed) provided in the accompanying table.

These 12 distances, along with an assumption of the location of the north star, provide a basic and useful vocabulary ${ }^{10}$.

[^5]
## DISTANCE CHARTS



Distance From
hash mark to hash mark boundary to the nearest outer hoop corner hoop to nearest interior hoop corners to the nearest corner hoop boundary to the nearest center hoop corner to corner - east to west corner to corner - south to north corner to distant horizontal hoop corner to distant vertical hoop corner hoops to the peg center hoops to the farthest outer hoops penalty spots to the nearest outer hoops penalty spots to the center hoops

Example Shown
Distance (in yards)
c1 to $1^{\text {st }}$ hash mark
south boundary to $h(1) \quad 7.0$
h(4) to h(5) 7.8
c2 to h(2) 9.9
south boundary to $h(5) \quad 10.5$
c2 to c3 28.0
c1 to c2 35.0
c4 to h(1) 22.1
c4 to h(3) 28.9
$\mathrm{h}(2)$ to the peg 12.6
h(12) to h(3) 18.8
western spot to $\mathrm{h}(8) \quad 12.6$
western spot to h(5) 15.7

## Your Critical Distance (CD)

Knowing the distance of a shot is one thing. Knowing your likelihood of delivering at that distance is another. Here the croquet world talks in terms of Critical Distance - CD - the distance at which you roquet a ball on average $50 \%$ of the time. In the tables that follow, we summarize data from Robert Fulford (Nottingham List, Jan 1, 2020) where he relates Critical Distances to clearance percentages at different lengths ${ }^{11}$.

We have chosen to break the data down into distances for "Initial Clearances", those that can occur as the first defensive shot of a turn, and distances for "Come-Back Clearances", those that can occur on a subsequent turn after an Initial Clearance fails, or, as in some of these new games, during the same turn.

|  | Initial Clearances |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| $\mathbf{C D}$ | $\mathbf{9}$ | $\mathbf{1 2}$ | $\mathbf{1 5}$ | $\mathbf{1 8}$ |
| $\mathbf{8}$ yards | $55.2 \%$ | 68.8 | 79.4 | 87.1 |
| 16 | 29.6 | 38.7 | 47.3 | 68.8 |
| $\mathbf{2 4}$ | 20.0 | 26.4 | 32.7 | 38.7 |


|  | Come-Back Clearances |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| CD | $\mathbf{9}$ | $\mathbf{1 2}$ | $\mathbf{1 5}$ | $\mathbf{1 8}$ |
| 4 yards | $87.1 \%$ | 95.7 | 98.9 | 99.8 |
| 6 | 68.8 | 82.3 | 90.8 | 95.7 |
| 10 | 45.6 | 58.2 | 68.8 | 77.5 |
| $\mathbf{1 2}$ | 38.7 | 50.0 | 60.1 | 68.8 |

The length of an Initial Clearance is often close to the distance from the last hoop to the Current one. These are easily determined from the charts presented above and can aid in the decision of whether to attempt a clearance or not. How hard to shoot is another important variable and depends upon lawn conditions and the likely length of a Come-Back clearance if you miss.

The length of a Come-Back attempt depends on the location of Current-Hoop on the lawn - is it a corner or a central hoops?
Corner Hoops: Many come-back clearance shots will be tried from the north or south Boundaries, or the east or west Boundaries that define a corner. The odd-numbered hoops are made "shooting-out" from the corner while the evennumbered hoops are made "shooting-in" to the corner. Thus, a target ball in front of an odd hoop will be closer to the boundary and closer to the clearing ball than it is for an even hoop. The difference can influence the probability of clearance.

[^6]Center Hoops: These are their own world. Initial clearance attempts to center hoops are usually short but come-back attempts can be quite long.

The importance of Short Hoops: The proximity of $h(5), h(7), h(11)$ and $h(1)^{12}$ to their previous hoops, $h(4), h(6), h(10)$, and $h(12)$ makes these the four shortest hoops. Play at them often determines the winner of games because they pose unique opportunities and have unique dangers. These short hoops can be further divided into two sub-categories: "Short" Hoops: $\mathrm{h}(5)$ and $\mathrm{h}(11)$, and "Short/Short" Hoops - $h(7)$ and $h(1)$ [from $h(12)]$. The difference in these is the length of a clearing shot by the ball that scores the preceding hoop. Making $h(4)$ or $h(10)$ can put Striker further away from $h(5)$ or $h(11)$ than does making $h(6)$ or $h(12)$ relative to $h(7)$ or $\mathrm{h}(1)$. The difference can be almost 5 yards -11.8 yards for the Short Hoops, and 7.0 yards for the Short/Short Hoops. To estimate the importance of this difference we used Fulford's calculations for hit-in probabilities as a function of CD to obtain hit-in fractions at 8 and 12 yards. On average, across CDs, there is a difference of approximately $18 \%$, a significant difference.

Another difference is that a firm center-ball clearance at a "Short" hoop can send the Oppo-Ball 13 yards away to the sideline while a firm center-ball clearance at a "Short/Short" hoop only goes 6 yards before becoming a boundary ball.

## Skills Tests to determine your CD

GC-based games rely on the ability to roquet/clear other balls from various distances, to lag balls to specific locations (in front of hoops, or blocking other balls), and to make hoops. Here are the tests that we use to measure these skills:

Roquets and Clearances: Shooting a predetermined number of balls at a hoop from specified distances and keeping track of the hit rate provides a simple way to measure this skill. Counting hoops-made gives further information concerning accuracy. We chose 6 and 17.5 yards as our distances -6 yards because it is a common secondary-clearance distance, and 17.5 because it is the average of the medium and long distances between hoops. Further encouraging these distances is that they are easy to identify -6 yards is the distance to $h(1), h(7), h(3)$, and $h(9)$ after marking-in one yard and 17.5 yards is the distance to $h(2)$ starting along the line between $h(1)$ and $h(2)$, level with $h(5)$. Similar distance points are easy to find shooting to $h(10), h(8)$, and $h(3)$. We chose 12 as the number of balls that players would shoot at each distance -12 is enough to develop a pattern without succumbing to boredom or fatigue.

Lagging and Shooting: These skills are combined into one test -our 1-Ball Drill. It involves playing a single ball starting from c4.

[^7]Striker gets two shots to attempt each hoop in the normal order that encompass a GC game: $h(1)-h(12)$ and then back to $h(3)$. Striker turns his attention to the next hoop after two shots, whether he makes or fails at the just played hoop, and without regard to where the ball goes. Balls are played as they lie. Scoring or jawsing a ball with the $2^{\text {nd }}$ shot to any hoop scores a single point. Scoring or jawsing on the $1^{\text {st }}$ shot to an even hoop scores two points, making the maximum possible score for the test 19 points - one point at each of the 7 odd hoops and two points at each of the 6 even hoops.

Jawsing: From Odd Hoops, the player gets two shots for the next even-numbered hoop. For example, jawsing h(1) leads to two shots for $h(2)$, starting from the ball's position in the jaws of $h(1)$.

From Even Hoops, Striker places a $2^{\text {nd }}$ ball near the even-numbered hoop, such that this ball can rush the ball coming out of the jaws to the next odd hoop. For example. If Striker jawses $h(2)$, then he places a $2^{\text {nd }}$ ball to the west and north of $h(2)$. Striker then plays the ball in the jaws. This should be a short-controlled shot that moves the ball from the jaws to a position that is rushable to the next odd hoop, $\mathrm{h}(3)$. Then, Striker plays a single shot with the placed ball rushing the other ball to the next odd hoop. No matter what the result, Striker then returns to playing his original ball, playing his $2^{\text {nd }}$ and final shot to the next odd hoop. If all has gone as planned the rush was successful - then the rush will leave a simple hoop attempt.

Going Half-Way: After his $1^{\text {st }}$ shot, Striker may conclude that a hoop is not-makeable. Striker should use his $2^{\text {nd }}$ shot to go half-way to the next hoop. However, going beyond half-way results in a trip to a penalty spot (the worst one please!).

Fearsome Foursome: We are grateful to the players for agreeing to take these skills tests and for letting us report the results.

|  | Hits/Hoops <br> @6 Yards | Hits/Hoops <br> @17.5 Yards | Score <br> 1-Ball |
| :--- | :---: | :---: | :---: |
| Sherif | $11 / 4$ | $5 / 0$ | 8 |
| Matthew | $12 / 2$ | $4 / 1$ | 11 |
| Danny | $10 / 4$ | $7 / 4$ | 9 |
| Steve | $11 / 6$ | $4 / 3$ | 7 |

These guys are good! Analyzing their results using Fulford's CD data suggests that they have CDs between 12 and 15 yards, which accounts for some of their success.

## Chapter 2



The $1^{\text {st }}$ shot of a turn can be used for any purpose available in traditional GC. In addition, if (and only if!) the $1^{\text {st }}$ shot involves a roquet, then a $2^{\text {nd }}$ shot is earned and can once again be used for any purpose. Game is to 5 . Hoops are played in conventional order except that the tie-breaker hoop at a 4-4 score is $\mathrm{h}(12)$. A Bamford-Start is used to minimize the benefit of going first.

The intrigue of 2-Shot is the $2^{\text {nd }}$ shot. Once earned, it adds new dimensions to play. Among these are the ability to use another ball as a "pioneer" to aid in offense or to use it as a "Stepping-Stone" to aid in defense.

## THE RULES OF 2-SHOT

The game of 2-Shot follows the rules of GC, except that AC rules apply for striking faults, and there are the following modifications:
I... A Turn: A turn involves one or two shots. A point is scored by Striker's team if Striker runs a hoop in order with his $1^{\text {st }}$ shot, or this $1^{\text {st }}$ shot causes another ball to run a hoop in order. If Striker's $1^{\text {st }}$ shot makes a Valid Roquet, including before or after scoring a hoop for either team, then Striker can play a $2^{\text {nd }}$ shot which can be used for any purpose, including hitting a ball previously hit, or scoring a point (or points) for either team. The $2^{\text {nd }}$ shot is optional. If taken, it is used to determine whether Striker is offsides. If not, then offsides is determined by the results of Striker's $1^{\text {st }}$ shot.
II... Marking-In: After the $1^{\text {st }}$ shot, and then again after the $2^{\text {nd }}$ shot of every turn, all balls that were sent out of bounds, and all nonStriker balls that were moved into the area that is within one 1-yard of a boundary line, are marked-in to the 1-yard line. If Striker remains in bounds after his $1^{\text {st }}$ shot, then his $2^{\text {nd }}$ shot (if any) is taken from Striker's current position, it is not marked-in. However, the Striker ball is marked-in to 1 -yard, at the end of his turn. Two or more balls may overlap during the mark-in process. Balls on the 1 -yard line that need not be marked-in are left in place. The other balls are placed touching each other along the 1-yard line (or lines if a corner is involved) in an order determined by Striker.
III... Legal Shots: As in AC, Croquet shots (i.e., when balls are touching) are legal shots and, when balls are in proximity, a hit into an object ball is considered a roquet which is an exemption to the striking fault of allowing multiple hits, thus double-hits are allowed.
IV... Order of the Hoops and Winning the Game: Game is to 5 points. Play starts at a randomly chosen hoop and then proceeds in numerical order wrapping around from $h(12)$ to $h(1)$ if necessary.
V... Valid Roquets: A Valid Roquet involves the Striker-Ball contacting another ball on the $1^{\text {st }}$ shot of a turn. However, a croquet shot played out of a corner is a Valid Roquet only if it involves an Oppo-Ball and not Partner. This prevents occupation of corners by two balls of the same team which could facilitate play on its next turn by the ball that entered the corner first.
VI...The Start of a Game: Balls enter from a new baulk line that starts one-yard north and one-yard west of c 4 and runs north for six yards along the east boundary. After u plays to start a game, the $r / y$ team can switch from being $r / y$ to being $u / k$ by claiming the play of $u$ as their own. The former $u / k$ team becomes the $r / y$ team and plays $r$ next, etc. This is a Bamford-Start.

## NOTES ON 2-SHOT

## New Roles

The possibility of a $2^{\text {nd }}$ shot allows balls to take on new roles in 2-Shot that are not seen in traditional GC.

Pioneer-Ball: Suppose your Partner-Ball is near the Current-Hoop but is not in position to make it. Your ball (or the Spent-Ball) can be positioned as a "Pioneer" for Partner. Under the rules of 2-Shot, on his next turn, Partner can roquet the Pioneer with his $1^{\text {st }}$ shot and then attempt the hoop with his $2^{\text {nd }}$ shot.

This Pioneer-Ball is also available to Oppos, and one of them plays after you and before your Partner. Therefore, placing a Pioneer to be useful to Partner and not Oppos is vital. Think "clear the Danger-Ball, and then set a Pioneer for Partner", or "rush Partner close to the hoop and then set a Pioneer for Partner", or "rush Spent to Partner and then clear Danger", etc.

Stepping-Stone ${ }^{13}$ : As in GC, long clearances are possible in 2-Shot and sometimes necessary. But there is an alternative when you and your Partner are a long way away from an Oppo-Ball that is in position at the Current-Hoop: If the rotation allows, you can become a Stepping-Stone for Partner. Clearance is then a two-step process: Partner roquets the Stepping-Stone and then shoots to clear the Oppo-Ball. Both of Partner's shots are shorter than a full clearance shot. The goal is to give your team a higher chance of moving the Oppo-Ball by making two shorter roquets than you have by trying to make one of your two long roquet attempts.

There are three primary positions to consider for a Stepping-Stone: (i) At a distance from Partner where he can likely roquet it on his $1^{\text {st }}$ shot and clear an Oppo-Ball on his $2^{\text {nd }}$ shot. (ii) Close to Partner so that he can attempt to "glance" off and proceed to the OppoBall at the same time. (iii) Very close to Partner so that your ball can be used in a double-hit.

Don't forget that Oppos will have a chance to disrupt your plans!

[^8]
## The Opening Turns

The play of the $1^{\text {st }}$ ball ( $u$ ) to start a game is unique for three reasons:
(i) Before $u$ plays there are no other balls on the lawn. Therefore, $u$ cannot roquet another ball to gain a $2^{\text {nd }}$ shot.
(ii) All balls enter the game from the Start-in Baulk-Line. From there, $u$ does not have a shot on $h(1)$. This is one time a ball does not have any chance (however remote) of scoring a point during a turn.
(iii) There is a "Bamford-Start" - after u plays, the $r / y$ team can claim the position of $u$ as their own (i.e., by switching colors) ${ }^{14}$. This format helps make the opening a more neutral event.

## The First Ball

Figure 2S-1 shows the Start-in Baulk-Line and eleven possible opening positions for $u$ where it is assumed that the $1^{\text {st }}$ hoop is $\mathrm{h}(1)^{15}$.
Three ( $1,2,3$ ) offer immediate hoop shots for $u$ at $h(1)$ on his next turn. Position 1 , slightly west of $h(1)$, and Position 2 slightly east of $h(1)$, have short hoop shots. Position 2 is riskier for $u / k$ than Position 1. In both cases $r$ can attempt to roquet $u$. But if $r$ hits $u$ at Position 2, then he will be moving both $u$ and $r$ closer to $h(1)$, perhaps facilitating a hoop shot. Whereas if $r$ hits $u$ at Position 1 , then he will be moving both balls farther from the hoop, probably precluding a hoop shot. Position 3 , on the baseline south of $h(1)$, has a long hoop shot, and can be useful (i.e., set up a short hoop shot) if a Pioneer-Ball can be positioned somewhere near to him. None of these positions should be set intentionally by $u$, and all should be taken over by Oppos using the Bamford-Start!

Positions 4 and 5 do not give $u$ immediate hoop shots. But scoring opportunities for $u$ can be developed if, after $r$ plays, $k$ lags to Pioneer Position, either on his initial shot or after roqueting u or $r$. Of these, Position 4 (to the west) is the most attractive as it is less likely to be disturbed by $\mathrm{r} / \mathrm{y}$. Again, we believe that $u$ should not intentionally seek these positions and that Oppos should take them over with the Bamford-Start.

[^9]

Position 6 is on the corner spot in c 1 and brings into play Rule V. It states that a croquet shot played out of a corner is always legal but is only a Valid Roquet - allows a $2^{\text {nd }}$ shot - if the croqueted ball is an Oppo, and not Partner. Here is why this rule was instituted:

Suppose $u$ shoots into $c 1$ and is marked-in to the corner spot. r takes his turn but does not roquet u or shoot into c 1 . Then k shoots intentionally into $c 1$, missing $u$. $k$ is marked-in along the 1 -yard line touching $u$ (on either side) at $k$ 's option. Here $k$ will choose to be to u's east. y plays next and goes somewhere. Once again, it is $u$ to play. In the absence of Rule $V$, $u$ could claim a Roquet by engaging in a croquet shot with $k^{16}$ that takes $u$ to position at $h(1)$. From here, $u$ could have a makeable hoop shot. This strategy is possible at every hoop and could lead to degenerate play, having the first ball of a team to a new hoop always go to the nearest corner - hence Rule V. This rule makes Position 6 less useful to $u / k$ and less desirable to the team with the Bamford-Start.

[^10]Positions 7 and 8 are both close to the Start-in Baulk-Line ${ }^{17}$ and present tices for $r$. Position 7 is on the south boundary, 3-4 yards west of $c 4$. If $r$ does not move $u$ (rush it away), then $k$ should be able to rush $u$ toward $h(1)$ and go to Pioneer Position for $u$. This can result in $u$ having an easy next turn - roqueting $k$ as a Pioneer to gain a hoop shot. Position 8 is within a foot of the corner spot for c4. Like Position 7, Position 8 can be rushed by $k$ to $h(1)$ and then $k$ could become u's Pioneer. But Position 8 affords an additional possibility - a double-hit by $k$ that sends $u$ and $k$ both to $h(1)$. At worst, $k$ can use his $2^{\text {nd }}$ shot to set-up as a Pioneer for $u$. At best, $k$ could have a hoop shot for himself as his $2^{\text {nd }}$ shot.
$r$ has strategic counters to positions 7 and 8 . Perhaps the best is to have $r$ use its $1^{\text {st }}$ shot to roquet $u$ gently such that $r$ gains a rush on $u$ to the north. Then, $r$ can drive $u$ away with his $2^{\text {nd }}$ shot, while trying to stay relevant (near the hoop) himself. As $r / y$, we would use our Bamford-Start to take over Position 8, but the decision on Position 7 would depend on how far it is from the Start-in BaulkLine, and our ability, or Oppo's, to gain the needed rush to destroy u.

Finally, Positions 9, 10, and 11 show three locations for $u$ that leave $r / y$ without a clear decision concerning the exercise of the Bamford-Start. None are in position to make $h(1)$, and none are sufficiently close to $h(1)$ for $k$ to become an effective Pioneer for $u$.

These positions for $u$ were carefully chosen such that $u$ is close to $h(1)$ - but not too close - which, in the end, is probably the best $u$ can do - leave it unclear whether r/y should use their Bamford-Start. Once the Bamford-Start is used, or not, one team will own these positions. We do not believe $r$ should shoot at $u$ in Position 9 or 10. They are both north of $h(1)-$ what can be gained? We think the most interesting option is position 11 - intended to be at r's critical distance - hence a 50/50 tice. r should shoot, hoping to skim off to $\mathrm{h}(1)$, or drive u away from a $50 / 50$ shot by k .

## The Second Ball and Beyond

In Figure 2S-2, we assume that u chose to go Position 11, the tice position, shown in Figure 2S-1. r responded with a firm shot at $u$ that missed. $r$ could have shot gently, hoping to glance off $u$ and gain position at $h(1)$. This would be a good result, but an unlikely one that could backfire by leaving $r$ as a possible Pioneer for $u$ or a useful double for $k$ !

Figure $2 \mathrm{~S}-3$ carries play two additional turns. k shoots gently at u and misses but ends in a position that may help u as a Pioneer.

[^11]k did not roquet on his $1^{\text {st }}$ shot and, therefore, does not get a $2^{\text {nd }}$. y shoots at u hoping to break up $\mathrm{u}^{\prime}$ s plan to roquet k and attempt the hoop. y chooses carefully how hard he hits such that he ends up as a Pioneer ball for r, but not for u or k .


The Geographic Divide: In Figure 2S-3, r and $y$ are to the west of the hoop while $u$ and $k$ are to the east. Each team has a Pioneer, that could help their team, but neither Pioneer helps Oppos. The possibility of using geography in this way - helping Partner and not Oppos - occurs frequently in 2-Shot and is something to watch for.

## Play at Hoops Other Than the $1^{\text {st }}$

As in GC, after a hoop is made, $h(1)$ in this case, the $1^{\text {st }}$ ball played to a new hoop, here $h(2)$, should seek position. In Figure $2 \mathrm{~S}-4$, y has made $\mathrm{h}(1)$ and u has taken his $1^{\text {st }}$ shot to $\mathrm{h}(2)$ and achieved position, ideally to a position where a long gentle roquet will not leave $r$ or $y$ an easy hoop shot.

From Figure 2S-4, In traditional GC, it is likely that $r$ shoots hard at $u$ and misses. $k$ plays next and would try to protect $u$ from $y$, but will probably fail. Then $y$ shoots hard at $u$, and likely misses. It will now be $u$ to play with an easy hoop shot. This chronology suggests that if the first ball played to a new hoop gains position and the shots at it by opponents are long, then it is likely that the team at the hoop will have control and should score the hoop.

But things can be different in 2-Shot. $r$ uses his $1^{\text {st }}$ shot to rush $k$ to the east, away from $y$, and then $r$ lags to a position to become a Stepping-Stone for $y$, Figure 2S-5, with the goal of having $y$ hit $r$ and then destroy $u^{18}$.

Setting and Using a Stepping-Stone

$r$ is a Stepping-Stone but $r$ is also a possible Pioneer for $y$. $k$ would like to interfere with $y$ 's ability to roquet $r$ and then clear $u$ or score the hoop. Given how far $k$ is from the action, there are no great options. Clearing $r$ or $y$, or making the hoop is unlikely. Instead, $k$ attempts to gain position at $h(2)$, somewhere between $u$ and $r$, Figure $2 S-6$. $k$ does this hoping that the position he achieves will block y from the hoop, and/or block y from u but it could backfire giving y a double target.

It is now y to play. In his first shot y roquets $r$ (closer to $k$ ) - not shown - but does not get a reasonable hoop shot. Fortunately for y , k did not succeed in blocking y from $u$. Therefore, with his second shot, y chooses to clear u to the north boundary and manages to leave Partner ( $r$ ) near the Spent-Ball ( $k$ ) in good position to score, Figure 2S-7.

We will leave it here for now but note that it is $u$ to play. $u$ will choose between a gentle 10 yard roquet at $k$ or that might earn a hoop shot, or a strong clearing shot at $r$ that is more likely to hit, but less likely to lead to immediate offense...

[^12]
## New Options with Jawsed Balls

In Figure 2S-8, u is jawsed in $h(1)$; it is $r$ to play. In traditional GC, $r$ might concede the hoop and go halfway to $h(2)$. Not so in 2-Shot! The $2^{\text {nd }}$ shot opportunity creates new ways to play against a jawsed Oppo-Ball:
(i) $r$ can play for the hoop by hitting $k$ and hoping to come to rest in jump position - not shown.
(ii) $r$ can try to ricochet off of $k$ to north of $h(1)$ and then clear $u$ from the jaws - not shown.
(iii) $r$ can clear $k$ and then lag to the north/east of $h(1)$ so that $y$ can use $r$ as a Stepping-Stone to clear $u$, Figure 2S-9.

It is k to play. The most likely result is that k shoots at y and misses, Figure 2S-10, and y follows by roqueting r and then driving u from the jaws (a firm shot is needed since u plays next), Figure 2S-11.

While unlikely and not shown, k can shoot at $\mathrm{u}, \mathrm{y}$, or r and have a good result from any roquet. That is: (a) k can shoot at u hoping to score $u$. If successful, then $k$ would still have a second shot and could lag to or shoot $h(2)$, (b) $k$ can shoot at $y$ hoping to score $u$ on his second shot, or $k$ can shoot at $r$ and then destroy $y$, leaving $u$ to score if $y$ misses.


## Miscellaneous

Here are three additional things to keep in mind with the game of 2-Shot:
Play at a Tie-Breaker Hoop: In GC, $\mathrm{h}(3)$ is the Tie-Breaker hoop and has its own mythology. In 2-Shot it is simply the $9^{\text {th }}$ hoop in order and may or may not be extraordinary. Figure $2 \mathrm{~S}-12$ is part of a game that started at $\mathrm{h}(1)$. u has just scored $\mathrm{h}(8)$ to make the score 4 to 4 . It is $r$ to play to start the tie-breaker $9^{\text {th }}$ hoop, which in this case is $h(9)$. As in GC, playing first to the final hoop it is still an advantage but, in many cases, there can be a more balanced and lively battle to secure the win, Figure 2S-13.


Intentionally Skipping an Available $\mathbf{2}^{\text {nd }}$ Shot: The Rules specify that a $2^{\text {nd }}$ shot is earned by a roquet, but its use by Striker is optional. If taken, then it is used to determine whether Striker is offsides. If not taken, then offsides is determined by Striker's $1^{\text {st }}$ shot.

In Figure $2 \mathrm{~S}-14, \mathrm{~h}(6)$ is being contested. It is u to play. In Figure $2 \mathrm{~S}-15 \mathrm{u}$ has cleared r and advanced to $\mathrm{h}(7)$ with his initial shot. u has various options for his $2^{\text {nd }}$ shot. One of them is to skip it - to pass! Then $r$ will play. If $r$ shoots and misses $k$, then $k$ should score at $h(6)$ and $u$ would not be offside at $h(7)$.

## Advancing Partner



Advancing Partner: In Figure 2S-16, u has just scored $\mathrm{h}(2)$. In Figure $2 \mathrm{~S}-17 \mathrm{r}$ has shot to $\mathrm{h}(3)$ and achieved position. It is k to play. $k$ knows that he needs to clear/block $r$ from scoring. $k$ has various options: (i) $k$ can shoot at $r$. However, this is a less than 50/50 shot. (ii) k can proceed toward $\mathrm{h}(3)$ as a Stepping-Stone for u . But y plays next and can rush u such that k is useless, or y can shoot at k , moving k out of Stepping-Stone position with his first or second shot, or (iii) $k$ can try to advance $u$. In Figure 2S-18, k roquets $y$; then, in Figure 2S-19, k advances $u$ to $h(3)$, perhaps giving $u$ a hoop shot, but at least giving $u$ an easy clearance shot on $r$. However, as shown, the good shot on $u$ may leave $k$ vulnerable to a roquet from $y$ followed by $y$ clearing $u$.

This play is available without a conveniently placed Oppo. $k$ could gently roquet the left side of $u$. With a bit of luck and skill, $k$ could promote $u$ with his $2^{\text {nd }}$ shot. With a lot of luck and skill, $k$ could end up very close to $u$ and intentionally double hit them both to $h(3)$.

## Data from the Videos

All of the videos available for 2-Shot had a starting hoop of $h(1)$ and were play to 7 points. We have since made the starting hoop random and reduced the winning score from 7 to 5 points. These change allow the players of 2 -Shot to experience the entire lawn over multiple games. Additionally, the shorter version maintains the drama of a game without dragging out its length.

We reviewed the 2-Shot match played by Sherif Abdelwahab and Jeff Soo with the goal of trying to understand/determine the importance of the $2^{\text {nd }}$ shot. There are many questions that can be asked. We present some of the data, and hope others will comment on how these compare to numbers that they get from playing 2-Shot or from results from their experience with GC.

This match consisted of three games. Jeff won the $1^{\text {st }}$ game $7-4$, Sherif won the $2^{\text {nd }}$ and $3^{\text {rd }}$ games 7-3 and 7-5 ${ }^{19}$. Here are some stats: Turns per score: This match involved play at 33 hoops. The fewest number of turns for a score at an individual hoop was 1 . There were also 3 points scored with 3 turns and 6 scored with 4 turns. The max number of turns to score a point was 34 ; there were also single points scored with $26,20,18$ and 17 turns. The median number of turns per point was 7 , a number we believe is more reflective than the mean.

One shot or two per turn? There were 286 turns. Of these, 168 ( $59 \%$ ) involved one shot and $120(41 \%)$ involved two. Removing the 33 shots that were $1^{\text {st }}$ shots to a new hoop - assuming that these were only one shot - modifies the percentages to 135 ( $53 \%$ ) to 120 (47\%) suggesting that, when given a choice, the players earned and used the extra shot about $1 / 2$ the time.

Did the turn that scored use one or two shots? Of the 33 scoring turns, $13(39 \%)$ involved a single shot while $20(61 \%)$ involved 2 -Shots suggesting that the $2^{\text {nd }}$ shot was of benefit to scoring.

Finally, did the team that played the $1^{\text {st }}$ turn to a hoop win it? Of the 33 hoops, 14 ( $42 \%$ ) were won by the team playing first while 19 ( $58 \%$ ) were won by the team playing second. This suggests that the $2^{\text {nd }}$ shot reduces the benefit of going first to a hoop, and in this particular match going $2^{\text {nd }}$ was a benefit.

[^13]
## Statistics and the 2 ${ }^{\text {nd }}$ Shot

Suppose $u$ has just gone to position at a hoop and $u$ is 20 yards away from $r$ and $y$ and that $r / y$ want to clear $u$. Should $r / y$ have $r$ shoot at $u$ and if that fails then have $y$ also shoot at $u$, or should $r$ set a Stepping-Stone for $y$ and leave it to $y$ to clear $u$ on his own? We do the analysis using the data from Fulford assuming that $k$ is positioned where he cannot cause mischief by intervening.

| Distance | $\mathbf{C D}=\mathbf{9}$ | $\mathbf{C D}=\mathbf{1 2}$ | $\mathbf{C D}=\mathbf{1 5}$ |
| :---: | :--- | :--- | :--- |
| 20 | 23.8 | 31.4 | 38.7 |
| 16 | 29.6 | 38.7 | 47.3 |
| 10 | 45.6 | 58.2 | 68.8 |
| 4 | 87.1 | 95.7 | 98.9 |

If both $r$ and $y$ are prepared to shoot at $u$, then the probability that one hits is: $\operatorname{Prob}[r$ hits $u]+\operatorname{Prob}[r$ misses $u] * \operatorname{Prob}[y$ hits $u$ ]. At 20 yards the resulting probabilities are [CD=9: 41.9\%], [CD=12: 52.9\%] and [CD=15: 62.4\%].

If $r$ sets a Stepping-Stone for $y$, then the calculation is: Prob [y clears $u$ ] = Prob [y hits $r$ as Stepping-Stone] * Prob[y hits $u$ after hitting $r]$. To do the calculations we assume that: (i) the Stepping-Stone ( $r$ ) is positioned perfectly along the line from $y$ to $u$, (ii) it is set at the desired distance, and that the ricochet of $y$ off of $r$ can be ignored. All that is left is to pick the distance of the Stepping-Stone from $y$. The most obvious placement for it is in middle between $y$ and $u$ - at 10 yards. Here the calculations are: [CD=9: 21.8\%], [CD=12: $33.9 \%$ ] and $C D=15: 47.3 \%$ ]. In all cases setting a Stepping-Stone produces an inferior result to simply shooting! And this result ignores the fact that if $r$ or $y$ hit on their own with their $1^{\text {st }}$ shot and hit, then they will have a $2^{\text {nd }}$ shot that would allow them to shoot to position. Sherif and Jeff may have realized this - they rarely set Stepping-Stones, preferring to shoot. ... ${ }^{20}$

[^14]
## The SHIP List for 2-Shot

There is a huge reward for Shooting a hoop or Hitting a ball on the $1^{\text {st }}$ shot of a turn in this game. This generally demotes interference and position to afterthoughts.

Shoot: If the Striker has a possible hoop shot, shooting the hoop is still the top priority with the consideration that if another ball has a better hoop shot, Striker can hit it gently and possibly earn an easier attempt and move the threatening ball if it is an opponent.

Hit: The rewards for a roquet (hit) and the risks of taking position at the hoop are so great and many that most turns will be an attempt to hit another ball. If there is a Pioneer-Ball, then Striker can hit it and hope to stop in position to run the hoop on her $2^{\text {nd }}$ shot. This style of scoring is very similar to the Ricochet game. Even from long distance, Striker may opt to shoot a soft or medium paced roquet in the hopes that the $2^{\text {nd }}$ shot may be a hoop shot. This style of hitting in can steal points from an Oppo who has great control of the hoop and actually resembles the long hit-ins from the American Rules game. Hitting the same ball, a second time, can achieve a 180-degree change in fortunes. If a Danger-Ball is between Striker and the near corner, then Striker may glance off of it in order to reorient before clearing that same ball to the far side of the court. This can also work for a ball very close to the hoop that is blocking the Striker's hoop shot. A gentle $1^{\text {st }}$ shot can reorient the balls so that the Striker can more forcefully clear Oppo without knocking them towards the hoop on the $2^{\text {nd }}$ shot.

Interfere: It seems interference is much less useful in 2-Shot. Blocking an Oppo's $1^{\text {st }}$ shot only mildly inconveniences them and may help them by shortening the distance for an initial roquet (like a Stepping-Stone). Attempting to smother a ball gives that ball a croquet shot or an intentional double tap which still awards a $2^{\text {nd }}$ shot. The only true interference is to play behind an opponent ball, close enough to hamper the $1^{\text {st }}$ shot. The hampered ball can still turn around and hit the obstructing ball, but in doing so , is wasting its $1^{\text {st }}$ shot without improving its position on the court. In order to block an Oppo's $2^{\text {nd }}$ shot, the Striker must take a speculative position guessing where the Oppo will stop after its $1^{\text {st }}$ shot and hoping to block the resulting $2^{\text {nd }}$ shot.

Position: There will still be turns where position at the hoop is the goal of the $1^{\text {st }}$ shot, i.e., the $1^{\text {st }}$ shot to a hoop. Sometimes a player will choose to shoot immediately to a Stepping-Stone or Pioneer position, but more often, these positions will be the backup plan for a strategically paced attempted roquet. The in-off hoop shot is now a much more consequential play. If the Striker makes the Current-Hoop off of a ball, this allows for a $2^{\text {nd }}$ shot to get position at the Next-Hoop. Players may attempt to place Partner behind the Current-Hoop and hope for a hoop and roquet for their partner. This would similarly allow a player to score the Current-Hoop and get position at the Next-Hoop with the possibility to score multiple hoops in-a-row.

## Chapter 3



The order of hoops is determined by random draw, there is no offsides. Play at a hoop continues until one team scores it with both balls. Teams earn one point for each ball that scores and an additional point (three in total) if both of their balls score while opponents score none. The winner is the first team to reach 8 points.

Balls that score continue to play in rotation but change roles, going from offense to defense. Scoring twice before your opponent scores once is extremely valuable.

## THE RULES OF DUELING DUOS

Dueling Duos follows the rules of GC, except that AC rules apply for Striking Faults, and there are the following other modifications:
I... Current-Hoop: Hoops are selected by random draw (without replacement) from a bag that starts with 12 blocks, one for each hoop. The $1^{\text {st }}$ hoop is drawn before the game starts. Subsequent drawings are done immediately after a hoop is "Finished" (Rule VI).
II... Clips: Clips are placed on the Current-Hoop before play to it commences and are removed as balls score the Current-Hoop.
III... 1-Yard Mark-in: Any ball that ends a turn out of bounds, or within 1 yard of a boundary line, is marked-in 1 yard and is in play.
IV... Start-in Baulk-Line: A 6-yard long "baulk line" runs along the east boundary. It begins one-yard north and west of c4. Balls enter the game from anywhere along it and, thereafter, are in play.
V... Start of the Game: A coin is tossed. The winner plays $1^{\text {st }}$ with $u$ to the Current-Hoop.
VI... Scoring Points and Finishing a Hoop: Each hoop generates 3 points. A team earns 0, 1, 2, or 3 points by scoring its hoop with 0,1 , or 2 of its balls. Scoring both balls while the other team scores neither earns a $3^{\text {rd }}$ bonus point, thus, 3 points in total. A ball continues to play in rotation after it has scored the Current-Hoop. A hoop is Finished when one team scores with its $2^{\text {nd }}$ ball.
VII... No Offsides: The offside rules of GC do not apply.
VIII... Legal Shots: Croquet shots (i.e., when balls are touching) are legal shots and, when balls are in proximity, a hit into an object ball is considered a roquet which is an exemption to the striking fault of allowing multiple hits, thus double-hits are allowed.
X... Winning the Game: The first team to reach 8 points wins.

## NOTES ON DUELING DUOS

The key elements of DD are: (i) The Current-Hoop is determined by random draw. (ii) Play continues at the Current-Hoop until one team has scored it with both of its balls, (iii) Balls that score continue to play in rotation after scoring, converting from offense to defense, (iv) A team earns one point for each ball it scores and an extra bonus point ( 3 in total) if they score with both balls before the other team scores at all. And (v) Game is to 8 and will involve no more than 5 hoops.

Initial Play to a New Hoop: Not knowing the location of the Next-Hoop until the Current-Hoop is completed makes it more likely in DD than in GC that balls will congregate around the Current-Hoop. Once the Current-Hoop is scored, the "next" Current-Hoop is determined by random draw and the next ball in rotation following the one that scored (or caused the score) will play.


The panel of figures above presents the opening of a game assuming that $h(11)$ is drawn as the initial Current-Hoop. u plays first. As in GC, u should play offense and shoot for position, shown done successfully in Figure DD-1. It is $r$ to play. If this were a game of GC, then $r$ would be advised to shoot at $u$. If $r$ fails to clear $u$, then $k$ would play, also attempting to find position or to block $u$ from a clearance shot from $y$. y would try to clear u , and if that fails then u would score, and the players would proceed to the Next-Hoop.

Attempting to clear $u$ with $r$ and then with $y$ if $r$ misses is a possible strategy in DD. It would be the obvious play if the state of the game is such that $u / k$ will win if either or both of $u$ and $k$ can score at the Current-Hoop. This can happen late in a game (see the discussion below) but, in general, in DD r/y need not clear u in order to prevail at the Current-Hoop. It is even possible that letting $u$ score first gives $r / y$ the overall advantage at a hoop.

In Figure DD-1, it is assumed that after u goes to position, $r$, $k$, and $y$ all shoot to position as well, Figure DD-2. Then u scores, Figure DD-3. It is $r$ to play. $r$ can score but if he does, then $k$ can score winning the hoop (2-1) for $u / k$. Therefore, $r$ does not score and instead $r$ clears $k$. Here, by assumption, $r$ maintains position. Figure DD-4. it is $k$ to play. $k$ attempts, but fails, to clear y, Figure DD-5. y scores, Figure DD-6, which accomplishes two goals for $r / y$ : First, the obvious one $-r / y$ score a point making the hoop score (1-1). But then the subtle one -y blocks $u$ 's access to $r$ by going through $h(11)$. Finally, $u$ tries a desperation jump shot and fails, after which r scores, Figure DD-7.

This example highlights important aspects of DD:
Clearing an opponent ball is good but clearing and holding position is much better. If $r$ fails to get position in Figure DD-4 and $k$ fails to clear y, Figure DD-5, y can still score, but $u$ will be able to move to a defensive position (ideally wired from y) from which to clear $r$ later. This shows that a ball that scores is temporarily in a weakened position that can be further weakened by being blocked access through the hoop if the opponents can also score.

If $r$, an offensive ball, is taken out of position when clearing the next ball to play, $k$ (the "danger ball") when $k$ is also an offensive ball, then $k$ can simply return to position. Yes, in this example $y$ can score, but if $u$ can see and clear $r$, then it is possible that $r$ misses $k$ and then k scores! This establishes that, unless there is no other option, it is usually not desirable for an offensive ball to clear another offensive ball.

## Unique Aspects of the DD Scoring System

The first team to reach 8 points or more is declared the winner.
Three points are allocated at each hoop. Each hoop can be won by one point $[(1,2)$ or $(2,1)]$ if each team scores once at the currenthoop with one ball before a $2^{\text {nd }}$ ball of one team makes the hoop as well. Or hoops can be won by three points $[(3,0)$ or $(0,3)]$ if a team scores both its balls before the other scores any. Since three points are allocated at each hoop, the combined score of the two teams is always divisible by 3 . Therefore, it is mathematically impossible for the score to tie at ( $8-8$ ) because 16 is not divisible by 3 . It is also mathematically impossible for a DD game to extend beyond 5 hoops during which 15 points will have been distributed. After 4 hoops (a combined score of 12 points) either there will already be a winner - a team will have 8 or more points, or the score will be tied at (6-6) in which case the $5^{\text {th }}$ hoop will be the deciding hoop - one team or the other will win (8-7) or (9-6).

## How Long Should a Dueling Duos Game take Relative to a GC game?

In GC, a game can be completed after just 7 hoops with a score of (7-0) or the game can take as many as 13 hoops if the score reaches (6-6) and is decided by a tie-breaker (7-6). In GC, hoops are only scored once so at a maximum there can be 13 times a ball scores during a game. In DD, a team can win by scoring (3-0) at 3 consecutive hoops in which case there will have been only 2 scores per hoop, and 6 scores in total. A team can also win by a score of (8-7). Here it is possible that each hoop was won (2-1) or (1-2). In this case there will have been 15 scores in total.

If duration of a game is measured in units of "scored-hoops", then the shortest DD game will be just slightly shorter than the shortest GC game ( 6 scores to 7 ), while the longest DD came can be a little longer ( 15 scores to 13 ).

## Can a Team Structure Play to Maximize its Prospects of Winning the Current-Hoop with a Particular Score?

Getting to (2-1): Suppose the score of the game is (6-6). Then a (2-1) score will win for either team. Here are two ways to get to a $(1,1)$ hoop-score if u plays first to the $7^{\text {th }}$ hoop of this game which, by chance, turns out to be $h(1)$.

(i) All four balls in Figure DD-8 shoot to position at $\mathrm{h}(1)$ the Current-Hoop from the previous hoop (which could have been any other hoop) with u playing first. Each team has two short hoop shots. It is $u$ to play for the second time at this hoop. u scores making the hoop-score ( 1,0 ), Figure DD-9. $r$ could make $h(1)$ evening the hoop score at $(1,1)$ but that would allow $k$
to score and give $u / k$ a $(2,1)$ victory at $h(1)$ and a victory in the game. Realizing this, $r$ clears $k$, Figure DD-10, $k$ shoots to clear y but misses, Figure DD-11; y makes h(1), Figure DD-12.
(ii) Three balls shoot to position at $\mathrm{h}(1)$, Figure DD-13. And then y shoots from the last hoop, assumed to be $\mathrm{h}(9)$, and clears $k$, Figure DD-14. $u$ and $r$ score, Figure DD-15. It is $k$ to play.

## Getting to (2-1)



In both Figures DD-12 and DD-15, the hoop score is (1-1) - one ball that has scored for each team. In addition, neither team has an obvious follow-on scoring possibility. Is one position more likely than the other to produce a $(2,1)$ score? If so, which team and why?

One possible answer comes from the literature on GC: "...in doubles play the side losing the toss should ensure that its senior player can precede the better player of the two opponents, thereby neutralizing as much as possible the greater threat", Golf Croquet Tactics, $2^{\text {nd }}$ edition, Michael Hague, 2005). In addition, most players of GC would argue that the team playing first, $u / k$, should have their strong player play first, as $u$. These thoughts taken together suggest that the best order of play in a GC game, where each team has a Strong (S) and a Weak (W) Player and Team \#1 wins the toss and goes first, is ( $u-S 1, r-W 2, k-W 1, y-S 2$ ).

Does this analysis carry over to DD? If so, what does it mean to be "Strong" or "Weak", especially if we assume that the players involved are all of equal skill? Clearly, balls that have already scored have different roles than balls that have not scored. Suppose
$u$ has scored, and the $r / y$ team has the ability to choose which ball ( $r$ or $y$ ) scores first. If $u$ is considered "Strong", then applying Hague's statement to DD would suggest that $r / y$ would prefer to have $y$ score as that would have $r / y$ 's then Strong Ball (y) play before $\mathrm{u} / \mathrm{k}$ 's already Strong Ball ( u ). Interestingly, the same order would be suggested if $u$ has scored and a ball that has scored is considered "Weak" ${ }^{21}$. One of us, Ben, agrees with this analysis.

But I am not convinced that this is the correct analysis for DD. Here is an argument that suggests that if $u$ has scored first, then team $r / y$ would prefer to score first with $r$, leaving $y$ to duel it out offensively with $k$, with $u$ and $r$ acting in "supporting" defensive roles:

Suppose one ball of each team has scored. On any subsequent turn, each of the remaining offensive balls have three possible actions: (1) Attempt the Current-Hoop, (2) Prevent the other team's offensive ball from scoring, and (3) Go to position at the Current-Hoop. Suppose the offensive balls are k and y . Now consider the play of k when y is in position at the Current-Hoop and k does not have an immediate hoop shot. k cannot go to position, but must take-out y . Now consider the play of y when k is in position at the Current-Hoop and $y$ does not have a hoop shot. $y$ can take-out $k$, or he can go to position and leave $k$ to $r$. Which action $y$ takes will depend on what $r$ might do (vis a vis $k$ ) and the probability that $r$ will be able do it (taking account of the potential interaction of $u$ ). But $y$ does not have to take out $k$. I believe that this freedom favors $r / \mathrm{y}^{22}$.

While this is an interesting debate (at least for us), we both believe that whatever advantage $\mathrm{r} / \mathrm{y}$ might have, if they can pick which of their balls scores after $u$ is small, and therefore, it is hard to imagine a scenario where $r / y$ would not take a hoop shot that is presented to it no matter for which ball.

The important factor may be in recognizing a possible disadvantage based on the order of play for those balls yet to score. For example, If it is determined that $r / y$ is at a disadvantage when $r$ and $k$ are the remaining offensive balls, then $r$ should be playing more aggressive hoop shots rather than waiting for k's advantage to manifest.

[^15]Getting to (3-0): Suppose the game is entering play at the $5^{\text {th }}$ and final hoop with the score (7-5) in favor of $r / y$. The only way $u / k$ can win is by scoring both $u$ and $k$ before $r / y$ scores either $r$ or $y$ thereby generating a hoop score of ( $0-3$ ) and a game score of (7-8).
What should $u / k$ do? $u / k$ should expect to lose! One way to measure the weakness of $u / k$ 's position is to compare the prospects of the two teams winning the hoop by assuming that each score is a $50 / 50$ proposition. $u / k$ must score first AND second, probability $(0.5 \times 0.5)=0.25$ while $r / y$ need only score first or second, probability $(0.5+0.5 \times 0.5)=0.75$. That said, each score should be treated individually, as independent events - both teams should step up and take their chances ${ }^{23}$.

[^16]
## Data from the Videos

Four games are shown in the videos. The first two were played in 2020 when the rules specified that winning required reaching 11 points, and that the winning team be ahead by two or more points at the end of the Current-Hoop. These games were fun, but too long!

The first was a runaway. Stephen and Sherif (as $u / k$ ) prevailed 11 to 3 over Danny and Matthew (as $r / y$ ). The game was notable in that $u / k$ swept ( 3 to 0 ) at the $2^{\text {nd }}$ and $4^{\text {th }}$ hoops and prevailed after 5 hoops. The impact of the win-by-two rule was evident in the second game where Danny and Sherif (as $u$ and k) prevailed in overtime 22 to 20!

Here is additional information we gleaned: Out of the total of 19 hoops that were played, 4 were won (3-0) while 15 were won (2-1). Of these 15,9 were ultimately won by the team that scored $1^{\text {st }}$. Of these 9,6 were won when the $2^{\text {nd }}$ ball to score followed the first in rotation (i.e., $u$ scored followed by $r$ and then $k$ ) while 3 were won by the first team to score when it was not followed by the next ball in rotation (i.e., u followed by y , followed by k ). This lends some evidence to Ben's view of the desired order of scoring.

We looked at (2-1) scores ( 12 from the $2^{\text {nd }}$ game) and measured how long it took on average to score the $1^{\text {st }}$ time -153 seconds, the $2^{\text {nd }}$ time -82 seconds and the $3^{\text {rd }}$ time -222 seconds ${ }^{24}$. Clearly the $2^{\text {nd }}$ hoop proceeded the fastest with double that time being spent on the $1^{\text {st }}$ score and nearly triple that time being spent on the $3^{\text {rd }}$ score. Clearly the period of time spent at a hoop when both teams have one ball through involved the most thought.

The second two videos were played in October 2021. It involved just two members of the Fearsome Foursome - Matthew and Stephen. We added these games to test the desirability of modifying the winning score to 8 from 11 with no requirement to win by two. The table below charts the play during these games. The hoops were drawn randomly so we identify the draw number (in order from 1-5, potentially) and then identify the hoop that was drawn. We allow for three scores at the hoop - the $1^{\text {stt }}, 2^{\text {nd }}$, and $3^{\text {rd }}$ and, in each case show, the ball that scored the hoop and how long in minutes: seconds it took from the last score. Finally, we show TeamScores at the hoop and the cumulative score of the game.

[^17]Game \#1: Matthew started the $1^{\text {st }}$ game scoring both of his balls resulting in a hoop score of (3-0). This put Stephen at an immediate disadvantage. He gamely won the next two hoops each (2-1) and could have tied up the game with a $3^{\text {rd }}(2-1)$ score at the $4^{\text {th }}$ hoop. However, Matthew prevailed with another (3-0) at the $4^{\text {th }}$ hoop, giving him the win (8-4). Besides great shooting, we witnessed a wonderful intentional double Oppo clearance, a lucky double hit that ran a hoop (double hits are legal in this game) and three amazing clearances of Oppo from the jaws, all done by Matthew ${ }^{25}$.Game \#1 took approximately 28 minutes.

Game \#1 (Matthew: ( $\mathbf{u} / \mathrm{k}$ ) and Stephen ( $\mathrm{r} / \mathrm{y}$ )

| Draw | Hoop | $1^{\text {st }}$ score | $\mathbf{2}^{\text {nd }}$ Score | $3^{\text {rd }}$ score | $\mathrm{u} / \mathrm{k}$ | $\mathrm{r} / \mathrm{y}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | $\mathbf{1 2}$ | $\mathrm{u}-1: 20$ | $\mathrm{k}-2: 20$ | xxxxx | $3 / 3$ | $0 / 0$ |
| $\mathbf{2}$ | 5 | $\mathrm{u}-3: 50$ | $\mathrm{r}-1: 10$ | $\mathrm{y}-9: 30$ | $1 / 4$ | $2 / 2$ |
| 3 | 3 | $\mathrm{y}-2: 50$ | $\mathrm{u}-5: 00$ | $\mathrm{r}-0: 20$ | $1 / 5$ | $2 / 4$ |
| $\mathbf{4}$ | $\mathbf{1}$ | $\mathrm{k}-1: 50$ | $\mathrm{u}-1: 00$ | xxxxx | $3 / 8$ |  |

Game \#2 (Stephen ( $u / k$ ) and Matthew: ( $\mathrm{r} / \mathrm{y}$ )

| Draw | Hoop | $1^{\text {st }}$ score | 2 $^{\text {nd }}$ Score | $3^{\text {rd }}$ score | $\mathrm{u} / \mathrm{k}$ | $\mathrm{r} / \mathrm{y}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 6 | $\mathrm{k}-2: 40$ | $\mathrm{u}-1: 00$ | xxxxx | $3 / 3$ | $0 / 0$ |
| 2 | 9 | $\mathrm{r}-1: 40$ | $\mathrm{u}-2: 00$ | $\mathrm{y}-1: 00$ | $1 / 4$ | $2 / 2$ |
| 3 | 3 | $\mathrm{u}-3: 30$ | $\mathrm{r}-0: 10$ | $\mathrm{k}-0: 10$ | $2 / 6$ | $1 / 3$ |
| 4 | 1 | $\mathrm{y}-2: 50$ | $\mathrm{k}-5: 50$ | $\mathrm{r}-1: 40$ | $1 / 7$ | $2 / 5$ |
| 5 | 10 | $\mathrm{x}-2: 00$ | xxxxx | xxxxx | $1 / 8$ |  |

Game \#2: Stephen turned the tables in this $2^{\text {nd }}$ game by making $h(6)$ from the southern Boundary for a (3-0) win at the $1^{\text {st }}$ hoop. Then traded (2-1) scores at the next two hoops [h(9) and h(3)] reaching a game score of (6-3) favoring Stephen. Matthew scored first at the $4^{\text {th }}$ hoop [at $\mathrm{h}(1)$ ] and had two opportunities to tie up the game at (6-6) with a (3-0) win but failed both times. Matthew was able to keep the game alive and force a $5^{\text {th }}$ hoop with a (2-1) win, making the score (7-5), still in favor of Stephen. Stephen was in a position to win with a single score, which he did with $u$ at the $5^{\text {th }}$ hoop, $\mathrm{h}(10)$, but not before Matthew attempted and almost succeeded on a clearance of $u$ by having $r$ attempt to jump over a well-placed, blocking $k$ ! Game \#2 took approximately 25 minutes.

Conclusion: We all came away believing that a simple first-to 8-points game was an improvement over the old 11 point game, win by 2. Yes, a (3-0) win at a hoop is more damning in the shorter, but it is not impossible to overcome and provides appropriate drama!

[^18]
## The SHIP List for DD

We divide the decision-making process for Dueling Duos into four segments: one for each possible score at the Current-Hoop: ( 0,0 ), $(0,1),(1,0)$, and $(1,1)$. In general, the process for each of these is the same as that for GC, appropriate adjusted for the score. The SHIP List of priorities still applies, but it is complicated a bit if Striker has scored the hoop and by the viability of the target balls.

Shoot the hoop: Balls that have already scored skip this step. For balls that have not scored there are new complications relative to GC involving the disadvantage of being on the backside of the hoop for the next turn (possibly blocked from being an effective defender). Striker may want to score gently in order to better defend this hoop in the future. There is less benefit in scoring odd hoops with momentum, as the Next-Hoop is yet to be determined by the random draw process.

Hit a ball: In addition to evaluating the order and position of each ball, the Striker must be aware of which ball is still to score the hoop. If the Striker-Ball still has to score, center ball clearances are even more valuable. When choosing how hard to clear the ball that just played (Spent-Ball) try not to put it in front of a hoop that may be chosen as the Next-Hoop.

Interfere with a ball: If the Striker has already scored, it is less dangerous as a hoop blocker and should consider cross-wiring from opponents more often. This means that even if Striker barely scored, going back through the hoop is less intimidating as a block and has the disadvantage of blocking Partner as well.

If the Striker can still score, smothering shots may significantly delay any hoop shooting and smothering a ball that has already scored is a bad scenario if they decide to remain in a tight position and smother the Striker.

Take Position: If the Striker has already scored, corner position is critical so that opponents cannot clear the ball farther away and make defense difficult.

## Chapter 4



Each turn involves two shots and a possible $3^{\text {rd }}$ which is earned if there is a roquet on the $1^{\text {st }}$ or $2^{\text {nd }}$. The balls enter the game on their color corner spot, with u to play first from c1. A hoop can made only after a roquet. There are no offsides. One point is earned at each hoop. Game is to 5 , win by 2 .

Scoring often begins with a long roquet on the $1^{\text {st }}$ shot that is followed by two others: a lag to the Current-Hoop, and a hoop shot. Failure to have an initial roquet usually leads to defensive play that seeks to isolate the Danger-Ball.

## THE RULES OF 3-SHOT

The game of 3-Shot follows GC rules, except that AC rules apply for striking faults, and there are the following modifications:
I... Turn: A turn can involve 1, 2 , or 3 shots. Striker can always play two shots if he wants to and earns a $3^{\text {rd }}$ if there is a Valid Roquet (see Rule V) on his $1^{\text {st }}$ or $2^{\text {nd }}$ shots. Scoring points (for either team) does not alter the availability of a $2^{\text {nd }}$ or $3^{\text {rd }}$ shot during a turn. There are no offsides ${ }^{26}$.
II... Scoring Points and Winning the Game: The first point of a turn can be scored by Striker for himself only after a Valid Roquet. Additionally, a point is scored for Oppos if any shot by Striker, roquet or not, causes an Oppo-Ball to run the Current-Hoop. The winner is the $1^{\text {st }}$ team to reach 5 points.
III... Marking-In: After each shot of every turn, all non-Striker balls sent out of bounds, or moved into an area that are with-in 1 yard of a boundary line, are marked-in 1 yard. If the Striker-Ball is sent out of bounds on any shot, then it is marked-in one yard. However, if the Striker-Ball remains in bounds after his $1^{\text {st }}$ or $2^{\text {nd }}$ shots, then his $2^{\text {nd }}$ or $3^{\text {rd }}$ shot is taken from Striker's current position. StrikerBall is marked-in when his turn ends. During the mark-in process, two or more balls may overlap. Balls on the 1-yard line that need not be marked-in are left in place. The other balls are placed touching each other along the 1 -yard line (or lines if a corner is involved) in an order as determined by Striker. Balls being placed in contact does not satisfy the need for a Valid Roquet prior to scoring.
IV... Legal Shots: Croquet shots (i.e., when balls are touching) are legal shots. And, when balls are in proximity, a hit into an object ball is considered a roquet which is an exemption to the striking fault of allowing multiple hits, thus double-hits are allowed.
V...Valid Roquets: A Valid Roquet involves the Striker-Ball hitting into and moving any other ball, or at least causing it to shake.
VI...The $1^{\text {st }}$ Hoop and the $1^{\text {st }}$ Four Turns: The $1^{\text {st }}$ hoop is determined by random draw. Then play proceeds in standard hoop order Play returns to $h(1)$ from $h(12)$ if necessary.

The balls are placed on their colored-corner spots $-u$ in $c 1, r$ in $c 2, k$ in $c 3$ and $y$ in $c 4-$ and play begins with $u$.

[^19]
## NOTES ON 3-SHOT

Summary: In this game a $2^{\text {nd }}$ shot is allowed each turn and a $3^{\text {rd }}$ is possible if there has been a roquet with the $1^{\text {st }}$ or $2^{\text {nd }}$. Importantly, a point can be scored by Striker only after a roquet.

Offense: There are two basic scoring strategies, each involves three shots ${ }^{27}$ : (i) Roquet, Lag, and Hoop attempt (RLH), and (ii) Lag, Roquet, and Hoop attempt (LRH) ${ }^{28}$. These are both applicable at any time, but the latter works best when a ball (Oppo or Partner) has been left - usually unintentionally - near the Current-Hoop.

Offense: Roquet, Lag, and Hoop (RLH)


Figure 3S-1 shows a starting position for RLH. The Current-Hoop is h(3), and u is to play. u's first thought, and often, but not always, the best strategy is offense - identify and attempt to roquet the closest ball, hit it and go make the hoop. (Another option - defense, is discussed starting at Figure $3 \mathrm{~S}-17$ below). Here we assume that $u$ shoots from the east boundary and succeeds in roqueting his Partner, k , Figure 3S-2.

[^20]In general, balls stay where they come to rest after a roquet. However, if any ball goes out of bounds, then it is marked-in 1 yard and play continues. If a non-Striker ball is within one yard of the border, then it is marked-in one yard. If Striker stays in bounds, then, as in AC, it is played from where it comes to rest, but is marked in at the end of the turn.
u's next job is to lag to position at the Current-Hoop, h(3), Figure 3S-3. This is followed by what we call the "lonely walk" - the trek to the ball now more or less (!) in position at the hoop. Finally, u attempts the hoop, and, in this case, makes it, Figure 3S-4.

## Offense: Lag, Roquet and Hoop (LRH)



Figure $3 \mathrm{~S}-5$ shows another starting position. Again, the Current-Hoop is $\mathrm{h}(3)$; u is to play. From this position u could follow the RLH Strategy - roquet $k$, lag to $h(3)$ and then attempt the hoop. However, given the position of $y$, it may be easier to proceed - using LRH. u lags to a position near y, (a bit north and east is good) , Figure $3 \mathrm{~S}-6$. u roquets y gently and gains position at $\mathrm{h}(3)$, Figure $3 \mathrm{~S}-7$. $u$ attempts $h(3)$, and scores the hoop, Figure 3s-8.

This last example was an illustration of $\underline{L R H}$ when both $\underline{R L H}$ and $\underline{L R H}$ were reasonable choices. However, if circumstances were different, then the decision might be different. If $u$ were further separated from $k$, then LRH would have a greater probability of success. And, if y blocks access to h(3), then LRH should be preferred.

Figure 3S-9 shows a situation where the previous ball (y) jawsed at $\mathrm{h}(3)$. u has various options. The most aggressive is RLH, roquet k , lag to position and then score $h(3)$ with a jump shot. This is risky because u could knock y through scoring the point for r/y instead of $\mathrm{u} / \mathrm{k}$. This is true when both balls go through, as in GC.

Here is an alternative: $u$ goes to a position on the non-playing side of $h(3)$, Figure $3 S-10$. u roquets $y$, knocking it out of $h(3)$, Figure $3 \mathrm{~S}-11$. u goes to a position close to k , Figure $3 \mathrm{~S}-12$. u does not score, but the immediate scoring threat of y is removed. Additionally, u and k are together, and r and y are separated.


Not shown is the possibility of attempting to half-jump y out of $h(3)$ from the non-playing side while having $u$ end in position at $h(3)$, ready to score h(3) with u's third shot. This is elegant but risky! Flying over and missing y entirely is possible and is end of turn, as there would not have been a roquet on the first or second shot.

In Figure 3S-13, y is once again jawsed in the Current-Hoop h(3), but this time it is $r$ to play ${ }^{29}$. $r$ lags to the playing side of $h(3)$, Figure 3S-14. r peels (rushes) y at $h(3)$ with his $2^{\text {nd }}$ shot, Figure $3 \mathrm{~S}-15$. Then $r$ shoots hard to $h(4)$ with his $3^{\text {rd }}$ shot, but misses to the south boundary, Figure 3S-16.

[^21]
## (y) Jawsed, Partner (r) to Play



Scoring on the $2^{\text {nd }}$ or $1^{\text {st }}$ shots: $r$ gets a $3^{\text {rd }}$ shot in the last example because $r$ completed a roquet on his $2^{\text {nd }}$ shot. That is, the availability of this $3^{\text {rd }}$ shot is unrelated to the outcome of the hoop. To carry this idea further, suppose r's turn started in Figure 3S14. If $r$ rush-peels $y$ with his $1^{\text {st }}$ shot, then $r$ will have had a roquet on his $1^{\text {st }}$ shot and still has two remaining shots. $r$ can continue his turn, lagging to $h(4)$ and perhaps scoring that hoop.

Defense: Finding the nearest ball, attempting a roquet, and then proceeding RLH is always possible, but may not be the best strategy. Figure $3 \mathrm{~S}-17$ repeats Figure $3 \mathrm{~S}-1$. Here $u$ recognizes that roqueting k is not a sure thing. If $u$ does not roquet, then he will be unable to score at $h(3)$ this turn, and additionally $u$ will have difficulty disrupting the play of $r$, the ball that plays next ${ }^{30}$. Important to $u$ 's decision is the knowledge that, if left unperturbed, $r$ will have an easy roquet on $y$ and an immediate RLS attempt to $h(3)$. With this risk in mind, $u$ decides against RLS and chooses to follow a different strategy - what we call the Basic Defensive Strategy (BDS). $u$ does not shoot the shortest shot (at $k$ ) but instead shoots at $r$ or y , Figure 3S-18.

If $u$ is lucky and hits-in: Among players of equal skill, if $u$ roquets $r$ or $y$, then, $u$ should attempt $h(3)$, ignoring the attempt at $h(4)$ that will follow for $r / y$ if $u$ scores $h(3)$ and also ignoring the attempt at $h(3)$ that will follow for $r / y$ if $u$ fails at $h(3)$. In both cases $r$ will have an easy roquet on $y$, but to prevent this, $u$ would need to separate $r$ and $y$, giving up his hoop attempt. The benefit of the lucky roquet would be lost. It is almost always better to lag to the current-hoop and take a look!

[^22]
## Defense: The Basic Defensive Strategy (BDS)



If $u$ does not hit-in: The expected result of the $1^{\text {st }}$ shot in BDS is that $u$ shoots at $r$ or $y$ and misses. Here is how BDS continues: Suppose $u$ shoots between $r$ and $y$, missing both balls and going out of bounds, Figure 3S-18. u uses a stop shot as his second shot to rush $r$ far away - south of y and k - Figure 3S-19. u lags to k with his third shot, ending his turn near Partner, and with Oppos separated, Figure 3S-20.

Needing to decide between shooting at the nearest ball - the offense of RLH - or shooting at more distant Oppos - the defense of BDS - is the major repeating theme of 3-Shot.

## The Opening Turns

3-Shot starts with each ball on its associated color corner-spot, Figure $3 S-21-u$ in $c 1, r$ in $c 2, k$ in $c 3$ and $y$ in $c 4$, with $u$ playing first. From his position in c1, $u$ can shoot at any of the other balls with the shortest shot being at y . The next three panels show three possible outcomes: $u$ hits $y$, $u$ misses $y$ to the north, $u$ misses $y$ to the south ending in $c 4$. In each case the $1^{\text {st }}$ hoop is assumed to be $h(1)$.

In Figure 3S-22, u has hit (roqueted) y sending it slightly north. u can now attempt $h(1)$ RLH: u lags to $h(1)$, Figure 3S-23. u shoots and makes h(1), Figure 3s-24.
u Roquets y


We start again in Figure 3S-21. This time $u$ shoots at $y$ and misses to the north, Figure $3 \mathrm{~S}-25$. $u$ needs a $2^{\text {nd }}$ shot roquet in order to earn a $3^{\text {rd }}$ shot. u uses his $2^{\text {nd }}$ shot to rush y south, seeking to move y out of c 4 , Figure $3 \mathrm{~S}-26$. This course of action prevents a second shot contact for another ball. Then $u$ joins partner near c3, Figure 3S-27 achieving a BDS leave, $r$ and $y$ separated, $u$ and $k$ together.

## u Misses y but ends in c4

This third panel also starts with Figure 3S-21. This time $u$ shoots at $y$ and misses to the south ending in c4, Figure $38-28$. From this position $u$ can consider his first shot a "lag" and proceed LRH because $u$ can be marked in just north of $y$ and touching it. Then a take-off shot can achieve the needed $2^{\text {nd }}$ shot "roquet" while sending $u$ to deep position at $\mathrm{h}(1)$, Figure $35-30^{31}$. A good hoop shot can score $h(1)$, Figure $3 \mathrm{~S}-31$.


Depending on what $u$ achieves, $r$ will take over from Figures $3 S-24$, or $3 S-31$ and be for $h(2)$. $r$ 's other possibility is that he will start from Figure $3 \mathrm{~S}-27$ and be for $\mathrm{h}(1)^{32}$.

[^23]
## Data from the Videos

Videos from three games of 3-Shot are available. The scores were 7-6 (Abdelwahab/Essick over Huneycutt/Morgan), 7-5
(Essick/Morgan over Abdelwahab/Huneycutt) and 7-5 (Essick/Huneycutt over Abdelwahab/Morgan). The games were close with the lead changing hands often.

Since these games were played, the rules were modified to have a game played to just 5 instead of 7 and to have a random starting hoop. We believe these changes will shorten games on average, and still give players a chance, over multiple games, to play at all hoops.

Turns per Points: There were 37 points scored with the following distribution of number of turns: (1:13, $2: 8,3: 6,4: 6,5: 1,6: 1,7: 1$, and 8:1). The mode was 1 and the median was 2 . There were single points involving 5, 6, 7 and 8 turns. Shots per Turn: 96 turns were played. 6 involved only 2 shots. The remaining 90 involved 3 shots.

Strategy used during 3-Shot Scoring Turns: Of the 37 scoring turns, 31 were RLH, and 6 were LRH. Strategy used during 3-Shot Non-Scoring Turns: Of the 53 non-scoring turns, 18 were RLM, where " $M$ " indicates a missed hoop attempt, 11 were RLD and " $D$ " indicates a defensive play caused by a bad lag, and 24 were MRD. Thus, these players made 37 out of 55 hoop attempts and successfully lagged to a hoop shot 48 out of 66 turns. They were forced to play defense only 11 out of 90 turns. These results are consistent with how the players performed in the skills tests.

## The SHIP List for 3-Shot

Shoot the hoop: Determine the easiest roquet and go for the RLH. If the shot is exceedingly unlikely and there is a ball close to your hoop, try the LRH. Otherwise...

Hinder your opponents: Try to hit an opponent for an RLH but ensure that a miss allows you to try the BDS to separate opponents and then...

Improve your Position: If you are left with one more shot, it is usually because you cannot score the hoop. Try to give your team a better leave so you may score on the next turn.

## Chapter 5



A team can play either ball each turn, but to score it must play both balls in the Period of the Current-Hoop (there is an exception for a ball that is peeled by opponents). There are no offsides. The team losing at a hoop always plays first to the next one. Hoops are played in normal order. Game is to a minimum of 5 points. Assuming $u / k$ plays first to start a game, then $u / k$ must win by at least 2 points, while r/y can win by 1.

Winning requires scoring when your team goes first to Current-Hoops and at least once when your opponents go first. This is difficult and often requires cunning and "gambling" to succeed.

## THE RULES OF AC-GC

AC-GC follows the rules of GC except that AC rules apply for Striking Faults, and there are the following other modifications:
I... Either Ball: A Team can play either ball each turn. The team losing at the Current-Hoop always plays first to the Next-Hoop and can do so with either of its balls.
II... Legal Shots: As in AC: croquet shots (i.e., when balls are touching) are legal shots and, when balls are in proximity, a hit into an object ball is considered a roquet which is an exemption to the striking fault of allowing multiple hits, thus double-hits are allowed.
III... Offsides: There are no offsides.
IV... Marking-in: Any ball within 1 yard of a boundary or out of bounds at the end of a turn is marked-in 1 yard and is in play.
V... The Period of a Hoop: Each hoop has a "Period". The h(1)-Period starts with the opening shot of the game and extends through the shot that causes $h(1)$ to be scored. The Period of any other hoop starts with the $1^{\text {st }}$ shot taken after the last hoop was scored and extends through the shot that causes the hoop in question to be scored.
VI... Scoring a Point (Or Points), The Both-Ball Rule (the BBR): The Current-Hoop (and the Next-Hoop) can be scored in the same stroke by a team for itself starting with the $1^{\text {st }}$ play of its $2^{\text {nd }}$ ball during the Period of the Current-Hoop. Peeling an Oppo-Ball scores a point (or points) for Oppos, no matter how many opponent balls have played in the Period of the Current-Hoop. Peeling Partner when it has not played during the Period of the Current-Hoop does not score a point or points.
VII... Start-in Baulk-Line: The Start-in Baulk-Line runs 6 yards along the east Boundary starting 1-yard west and 1-yard north of c4.
VIII... Winning the Game: $u / k$ to 5 , win by 2 . r/y to 5 , win by 1. Play returns to $h(1)$ after $h(12)$ as needed.
IX... Start of the Game: A coin is tossed. The winner can play first with $u$ or $k$ to $h(1)$ or have Oppos do it.
X... Clips for Balls not Yet Played to a Hoop: After the third and fourth turns in each new Period, if a team does not play its $2^{\text {nd }}$ ball, then the clip for that ball is placed on the Current-Hoop and removed when that ball is played, or when that hoop is scored.

## NOTES ON AC-GC

The goal with AC-GC was to create a game that has no offsides, allows either ball of a team to play each turn, and has rules that negate the value of winning the coin toss. We started with the following changes to the rules of GC:

AC Order of Play: Teams alternate turns and can play either ball each turn.
AC Mark-in:

## No offsides:

All balls are marked-in (following AC rules) at the end of each turn.
A ball can be played to anywhere on the lawn without penalty.
These changes are necessary to define the game we were seeking but they are not sufficient to keep it from being trivialized. The most obvious problem occurs if the first ball to a hoop ( $u$ ) gains position and ror y fail to clear. Without further structure (some rule involving k , u can play the $2^{\text {nd }}$ shot of the $\mathrm{u} / \mathrm{k}$ team to score.

Here is another issue: Figure AC-1 shows a possible start to an AC-GC game. With no offsides, each ball shoots to position at a different hoop! (For the moment assume that the players have the skill to lag to position at their hoop in one shot). Then, on their next turns, each ball makes their hoop! Figure AC-2.


Clearly, more structure is needed. Here are our two initial additions:

The Period of a Hoop:

The Both-Balls Rule (BBR): A team cannot score at the Current-Hoop until both of its balls play during the Period of the CurrentHoop. The $2^{\text {nd }}$ ball of a team playing for the $1^{\text {st }}$ time during this Period can score. Additionally, a team can score at the Current-hoop if opponents peel them, even if the team had not satisfied the BBR ${ }^{33}$.

Now the start might play out as follows: Balls still choose to play in the $h(1)$-Period going to separate hoops, Figure AC-1, Then it is $u / k$ to play. u makes $h(1)$, having satisfied the BBR with $k$ and ending the $h(1)$-Period, Figure AC-3. As the losing team at $h(1), r / y$ go first in the $h(2)$-Period. Neither ball has played in this Period, so $r$ cannot score yet. r/y needs to play $y$ as its $1^{\text {st }}$ ball in the $h(2)$-Period if it wants $r$ to score immediately thereafter. Therefore, $r / y$ exercises its right to play either ball and has y play. But $y$ was "happy" at $h(4)$ and therefore "wastes" a turn by repositioning (or passing), leaving Figure AC-3 essentially unchanged. Then it is $u / k$ to play. With $k$ in position at $h(3), u / k$ has $u$ play to $h(5$. It is $r / y$ to play. $r$ scores $h(2)$ satisfying the BBR for the $h(2)$-Period and ending the $\mathrm{h}(2)$-Period, Figure AC-4. This is playing AC-GC as a non-contact sport!


AC-5

Such behavior is possible under the Rules of AC-GC, but it is not a necessary result of them. Eventually $r / y$ should realize that sending $y$ to $h(4)$ to start the game, only to have him pass on his $2^{\text {nd }}$ turn, is inefficient and undesirable $-r / y$ might send $y$ to $h(3)$ instead,

[^24]Figure AC-5. After u scores $\mathrm{h}(1)$, y can be the first ball to play during the $\mathrm{h}(2)$-Period and use his shot to clear k from $\mathrm{h}(3)$. k may catch on and decide to contest $h(2)$ instead of going to $h(3)$ in later games, resulting in Figure AC-6. Finally, $r$ may decide to fight for $h(1)$ instead of going to $h(2)$, with $k$ and $y$ following suit, Figure AC-7. In this case, there are four balls battling it out at $h(1)$. If this proves to be optimal behavior at all hoops then, while written into the rules, the freedom to go offsides, would be of no value. But that is not the case! As we will see, there are some ball configurations where having all of them at a single hoop is optimal, but each of these alternative scenarios has value as a Gamble for control of future hoops. Choosing how to deploy your troops is the fundamental challenge of $A C-G C$ and is discussed in depth in the rest of these notes.

Removing the Benefit of the Coin Toss: Going $1^{\text {st }}$ to start a GC game has value and can lead to a $5-4$ victory wherein the team playing $2^{\text {nd }}$ does nothing wrong except lose the coin toss. Our solution is embodied in the Starting and Winning Rules.

Starting: $\mathrm{u} / \mathrm{k}$ always play first to start a game ${ }^{34}$. The winner of the coin toss can play $\mathrm{u} / \mathrm{k}$ or have the other team play $\mathrm{u} / \mathrm{k}$.
Winning: The game is won by the first team to: (i) win at least one more hoop when Oppos go first than opponents win when you go first, and (ii) accumulate at least 5 points. This means that $u / k$ must win by two at least points while $r / y$ can win by one! (5-3) is a win by $u / k$, while ( $5-4$ ) is a win by $r / y$, etc. It might seem that being $u / k$ is a bad deal. Winning by 2 seems harder than winning by 1 . That would indeed be true if, as in tennis, teams alternated starting at hoops (serving), regardless of the outcome of the previous hoop (game). However, in AC-GC as in GC, the loser at the Current-Hoop always plays first to the Next-Hoop. This is what makes my rules fair. Here is an example:

The rules have $u / k$ play first to $h(1)$. Suppose $u / k$ wins the hoop. The rules also specify that the loser at a hoop always plays first to the Next-Hoop. Thus, r/y plays first to $h(2)$. Suppose $r / y$ lose at $h(2) . u / k$ has satisfied the requirement to win a hoop when $r / y$ goes first by winning at $h(2)$. Suppose the teams alternate hoops thereafter with $r / y$ playing first to $h(3)$. The result will be a (5-3) win for $u / k$. $u / k$ will have won 4 hoops when they went first (losing none) and 1 hoop when $r / y$ went first.

Suppose instead $u / k$ loses at $h(1)$. By winning $h(1), r / y$ satisfies the requirement to win a hoop when $u / k$ goes first. Suppose the teams alternate hoops thereafter with $u / k$ playing first to $h(2)$. The result will be a $5-4$ win for $r / y$. On net, $r / y$ will have won all 4 hoops when they went first and 1 hoop when $u / k$ went first.

[^25]Of course, $u / k$ can also win (5-0), (5-1), and (5-2). If the score reaches (4-4), (5-5), etc. then $u / k$ will win if the score reaches (6-4), (7-5), etc., with $u / k$ ahead. Likewise, $r / y$ can also win (5-0), (5-1), (5-2) and (5-3). If the score reaches (5-5), (6-6), etc. then r/y will win if the score reaches (6-5), (7-6) etc., with $r / y$ ahead. No matter how it happens, the winner will have won more times when the loser played first than vice versa.

These rules are fair and encourage the risk-taking behavior (described later) needed to win two hoops in a row that we call Gambling.
The remainder of these notes: (1) Introduces the basics of the game. (2) Describes Battles that play out primarily at the CurrentHoop. (3) Explains Gambles that seek to win two hoops in a row.

## THE BASICS

Notation: We use the following notation: T1-B1, T1-B2, T2-B1, and T2-B2, where "T" stands for Team (either u/k or r/y) and " B " stands for Ball. " 1 " and " 2 " identify going $1^{\text {st }}$ or $2^{\text {nd }}$ during the Period of the Current-Hoop either as a team or as a ball within a team.

Each hoop begins with play by T1-B1 and T2-B1 - the first balls of each team to play in the Period of the Current-Hoop. Then it is T1 to play again. At this point, T1 has a decision to make: Should it play its $2^{\text {nd }}$ ball, (T1-B2) or should it have the $1^{\text {st }}$ ball (T1-B1) play again? In general, T1 will play its $2^{\text {nd }}$ ball because it satisfies the Both-Ball Rule - the BBR - allowing T1 score on this or any subsequent turn. After that, it is T2 to play again. If T1 did not score, T2 must decide if it should play its $2^{\text {nd }}$ ball (T2-B2) or play its $1^{\text {st }}$ ball (T2-B1) again. This is often the fundamental decision point for the play of a hoop. Depending upon the action of T2, victory at the Current-Hoop will follow one of three patterns of play: A Quick Score, a (2-on-1) Battle, or a (2-on-2) Battle.

1. Quick Scores: There are three paths to a Quick Score - one involving only two turns by T1, and two involving three:
a. $\mathbf{2}^{\text {nd }}$ Turn Score: T1 plays first during the period of the Current Hoop with T1-B1. T2 plays T2-B1. Then T1 scores the Current-Hoop with T1-B2 (simultaneously satisfying the BBR). This is a " 2 nd Turn Score". When it happens, the Battle at the Current-Hoop can be over before the $4^{\text {th }}$ ball, T2-B2, has a chance to enter the fray! A $2^{\text {nd }}$ Turn Score can occur at all even numbered hoops where it is the result of a long and successful hoop attempt by T1-B2, shooting from its position around the Previous Hoop. In more advanced play, a $2^{\text {nd }}$ Turn Score no longer needs to be a "Hail Mary" and becomes possible at both even and odd hoops.
b. $3^{\text {rd }}$ Turn Score: There are two paths to a $3^{\text {rd }}$ Turn Score:
(i) Sending 2 Balls to Position at the Current-Hoop: Suppose T1 sends T1-B1 and T1-B2 to position at the Current Hoop. If T2 fails (or does not attempt) to clear or block both balls, then T1 should win the hoop on its next (3rd) turn (unless they blob a simple hoop shot!), which can be taken by either ball.
(ii) One Ball to Position and Two Misses by Oppos: Suppose T1-B1 shoots to position during the Period of the Current Hoop, T2-B1 fails to clear, T1-B2 does not get position, and T2-B2 (or T2-B1) attempts to clear T1B1 but fails. T1-B1 can now score on T1's $3^{\text {rd }}$ turn of the Current Hoop. Attempting a clearance with T2-B2 is always possible but is most relevant at the short hoops (h5, h7, h11, and back to h1).
2. (2-on-1) Battle: After the first 3 turns are taken during the Period of the Current Hoop (played by T1-B1, T2-B1 and T1-B2), T1 may have succeeded in getting one, but only one, of its balls to position. T2 may decide that a clearance attempt by T2-B2 is unlikely to succeed and choose instead to start a (2-on-1) Battle. This is what happens when T2 keeps T2-B2 out of the hoop temporarily and instead plays T2-B1 again. In this case, T2 becomes the " 1 -ball team" and T1 becomes the " 2 -ball team". A (2-on-1) continues until the 2-ball team scores the hoop, or the 1-ball team is successful in bringing in its $2^{\text {nd }}$ ball, T2-B2, which then starts a (2-on-2) Battle.
3. (2-on-2) Battle: If all four balls have played during the Period of the Current-Hoop, then there is a (2-on-2) Battle. This can happen immediately (with all balls playing at their first opportunity during the Period of the Current-Hoop), or it can be the result of a victory in a (2-on-1) Battle by the 1-ball team that enables it to bring in its $2^{\text {nd }}$ ball.

Summary of the Competing Interests: The team that plays first in the period of the Current-Hoop (T1) hopes to secure a victory at the Current-Hoop with a Quick Score. If that fails, then they hope to prevail in a (2-on-1). And, if all else fails, they want to win the (2-on-2). On the other hand, T2 hopes that a Quick Score is avoided (either T1 fails to get it or T2 prevents it) forcing a (2-on-1). Then T2 hopes to prevail in the (2-on-1) advancing it to a (2-on-2), and to then win that battle by scoring the hoop-point.

## Clarifying Examples

Two Clearances Generate a (2-on-2): Consider Figures AC-8 to $\mathrm{AC}-11$. The Current Hoop is $\mathrm{h}(1)$. All balls enter the game from the Start-in Baulk-Line. In Figure AC-8, u(T1-B1) goes to position at $h(1)$. In Figure AC-9, r (T2-B1) shoots and clears u. In Figure AC-10,
k (T1-B2) goes to position at $\mathrm{h}(1)$. In Figure AC-11, y (T2-B2) clears $k$. All four balls are in play at $\mathrm{h}(1)-$ this begins a (2-on-2). These figures shows that T2 can almost always force a (2-on-2) if T2 is able to uses its two balls to make consecutive clearances of T1 balls. $3^{\text {rd }}$ Turn Quick Score: In Figure AC-12, again starting from the Start-in Baulk Line, u shoots to position. $r$ tries to clear, but fails. Then $k$ attempts but does not get position. $y$ then misses another clearance attempt on $u$. 11 has only one ball (T1-B1) in position, but this should produce a $3^{\text {rd }}$ Turn Quick Score.

T1 fails to get position with either T1-B1 or T1-B2: Figure AC-13 shows a situation where T1 does not get to position with either of its balls ( $u$ and $k$ ). This allows both opponent balls $r$ and $y$ ( $T 2-B 1$ and $T 2-B 2$ ) to play and starts a ( $2-o n-2$ ). T2 will have the advantage if one of its balls is in position. Furthermore $\mathrm{r} / \mathrm{y}$ should win the hoop if both are in position, even though $\mathrm{u} / \mathrm{k}$ went first. Getting balls to position is critical in AC-GC.

$\mathbf{u}(\mathbf{T 1 - B 1 )}$ is in Position. Should r (T2-B1) Try to Clear or go to Position? This question can be broken down into two related question:
(i) How long is the initial clearance shot for $r$ at $u$ ? and (ii) If $r$ misses $u$, then how long is $r$ 's come-back clearance shot at $u$ ?

In Figure $\mathrm{AC}-14, \mathrm{u}(\mathrm{T} 1-\mathrm{B} 1)$ plays to position at $\mathrm{h}(1)$ for T1. r (T2-B1) can shoot at u and clear it, as shown in Figure $\mathrm{AC}-15$. This would be good thing for $\mathrm{r} / \mathrm{y}$ (T2) because it prevents T1 from immediately controlling the hoop. However, the clearance attempt is not a sure thing. It is long, approximately 21 yards, and, as such, missing, as shown in Figure AC-16, is the expected result. Should $r$ try to clear or just shoot to a spot near to $u$, as shown in Figure AC-17?

Attempt an Initial Clearance or just go near to T1-B1?


The answer to this question depends on the fraction of times $r$ succeeds with an initial clearance attempt (at the specified distance), and the fraction of times he makes come-back clearance attempts (at its specified distance).

Using data from the Appendix, and assuming $C D=12$, our calculations reveal a strong bias for $r$ to attempt the initial clearance. This bias still exists when $C D=9$, but less so. At still lower $C D$ 's, $r$ will want to cozy-up to u instead of attempting the clearance. These calculations suggest that, just as in regular GC, the more talented the players, the more likely it is that an AC-GC game will be a shooters game.

## BATTLES FOR THE CURRENT-HOOP

## The (2-on-2) Battle

The two primary Battle types, the (2-on-2) and (2-on-1), were outlined above. They and their "derivatives" are analyzed here.

Consider Figure AC-18. It shows an on-going (2-on-2) at $\mathrm{h}(1)$. The game has begun - it is in the $\mathrm{h}(1)$-Period. Starting from the Start-In Area, $u / k$ and $r / y$ have each sent two balls to the vicinity of $h(1)$ - hence the two " 2 ' $s$ " in the battle notation. Both teams have met the BBR which means that all balls can score $h(1)$ with their next shots. This is the definition of a (2-on-2).


In the abstract, a (2-on-2) gives each team a 50/50 chance of winning, but the odds shift as play proceeds depending on the positions of the balls and which team plays next. Further notation identifies some of these subtleties by naming Battles that are "derivatives" of a ( 2 -on- 2 ). These are the ( $2-$ on- $1^{*}$ ), ( $1^{*}$ on- $1^{*}$ ), and ( $1^{*}$-on- $0^{*}$ ).

In Figure AC-18, $r$ and $k$ do not have makeable shots at $h(1)$, but $u$ has an angled shot and $y$ has a straight-on shot. Suppose it is $r / y$ to play. A good player of $y$ could drive $u$ toward $c 1$ depriving $u$ of a hoop shot. But that would not gain much for r/y because $u$ would still be in the vicinity of $h(1)$. Instead, y should take the 6 -yard hoop shot. If y scores, Figure AC -19, then $\mathrm{r} / \mathrm{y}$ collect a point and $\mathrm{u} / \mathrm{k}$, as the loser to $h(1)$, will play first to start the $h(2)$-Period.
(2-on-1*): In Figure AC-20, y shot firmly at $h(1)$ and missed, glancing off the right upright, ending up near $h(6)$. It is $u / k$ to play. $u$ can take the angled hoop shot, or $u / k$ can shoot to position at $h(1)^{35}$. Suppose $k$ shoots to position, Figure AC-21. It is now $r / y$ to play. In GC, it would be y to play (because y follows $k$ in rotation). In AC-GC, r/y can play either ror y . Given $\mathrm{y}^{\prime}$ s distance from $h(1)$, he is unlikely to clear or block $k$. Therefore, $r$ should play, and clear $k$. $y$ has been temporarily "benched" because of his distance from the action. In cases like this, the Battle changes from a ( 2 -on- 2 ) to ( $2-\mathrm{on}-1^{*}$ ). The " $1^{\prime \prime}$ indicates that $r / y$ have only a single ball "around" the Current-Hoop, $h(1)$. The asterisk says that this ball can score - that is, $\mathrm{r} / \mathrm{y}$ has met the BBR ${ }^{36}$. A ( 2 -on- $1^{*}$ ) gives the

[^26]1* team less than a $50 / 50$ chance of winning because it must fight off two balls. In our estimation, for good players, it becomes approximately 55/45 in favor of the 2-ball team, u/k here.
(1*on-1*) and (1*-on-0*): In Figure AC-22, $r$ clears $k$. If $r$ stays near the hoop, then $u$ will likely play for $u / k$ because of u's proximity $^{\text {( }}$ to $r$. What follows will be a battle between $u$ and $r$, with both $k$ and $y$ "sitting it out". This is a ( $1^{*}$-on- $1^{*}$ ). It gives both teams a 50/50 chance of winning. Getting from (2-on-1*) to ( $1^{*}-\mathrm{on}-1^{*}$ ) is a good result for $r / y$. However, it could have gone badly for them if $r$ had mishit his shot, as shown in Figure AC-23. This results in a ( $\left.1^{*}-0 n-0^{*}\right)-u / k$ has a single ball around the hoop, while $r / y$ has none, the $\left(^{* \prime s}\right.$ ) indicate that both teams have met the BBR. A (1*-on-0*) gives the $1^{*}$ team a big advantage - perhaps 67-33, or more.

In Figure AC-23, $u / k$ has a decision to make: (i) have $u$ attempt the hoop, or (ii) have $u$ (or $k$ ) shoot to position at $\mathrm{h}(1)$. Our pick is to return $k$ to position. $r$ probably, and $y$ possibly, will shoot at $k$. If $r / y$ succeeds in clearing $k$, then $u / k$ can still have $u$ shoot the hoop, or $\mathrm{u} / \mathrm{k}$ can send one of u or k back to position. This illustrates the natural progression of play/thought in a (2-on-1*).
y shooting firmly and failing to make $\mathrm{h}(1)$ created the situation shown in Figure $\mathrm{AC}-20$. If y had shot gently and missed, Figure $\mathrm{AC}-24$, then both $r$ and $y$ would still be relevant at $h(1)$, leaving a ( 2 -on- 2 ). This observation might change $u / k$ 's strategy - $u$ could take the angled shot or clear $r$. The point is, there is a role for firm shooting, but the cost of missing can be greater in AC-GC than in GC.

Let's go back to Figure AC-18, this time assuming it is $u / k$ to play. We think $u$ should play ${ }^{37}$. $u$ can take the angled hoop shot at $h(1)$, but we recommend that u clear y instead ${ }^{38}$. Ideally, $u$ will send $y$ a long way away from $h(1)$ with a stun shot while leaving itself with a hoop shot, Figure AC-25. It is now $r / y$ to play. It is unlikely that $y$ will be able to prevent $u$ from taking the hoop shot and therefore $r$ should play again treating this as a (2-on-1*). $r$ 's best play is to block $u$ and then fight it out with $k$. $r$ should try to stay south and west of $k$ so that when $k$ clears $r$, $r$ will still be in the neighborhood of $h(1)$ and can block $u$ again. If $r / y$ "thinks" that $u$ will not make/take the hoop shot, then $y$ can return to $h(1)$. If $u$ in fact tries the hoop and misses, then the (2-on-2) or the (2-on-1*) Battle for a miss that leaves u far away - resumes. Figure AC-25 shows u executing a stun shot and getting position. If u does not get position, then $y$ should come back, reinstating the (2-on-2). It is also possible that the attempted stun shot fails, Figure AC-26. In this case, $y$ can shoot back to $h(1)$, putting $r / y$ in the driver's seat, now fighting a (2-on-1*). And so it goes...

[^27]

Conclusion: Play at a hoop in AC-GC often involves a (2-on-2) where both teams have satisfied the BBR. (2-on-2)'s are won, on average, by the team with better basic skills - hoop-shooting, blocking, clearing, etc. - where execution is tempered by the knowledge that both teams can play either ball every turn and that a ball that is out of the vicinity of the Current-Hoop loses value.

In the Beginning: Before introducing the other main Battle, the (2-on-1), it is useful to explore how the ability to play either ball combines with the BBR to affect play at the beginning of each hoop. Figures AC-27 and A8-28 could have come from a GC or an AC-GC game. u played first shooting for position at $h(1)$. It is $r$ to play. What should $r$ do? To answer this question, $r$ needs to know whether or not $u$ found position at $h(1)$. Depending on how that information is processed in each game, what strategy r/y adopts, can differ.

In GC: If $u$ finds position, and has an easy hoop shot, Figure AC-27, then $r$ will shoot at $u$, hoping to clear - succeeding perhaps $1 / 3$ of the time - Figure AC-29, but expecting to miss - perhaps $2 / 3$ of the time - Figure AC-30. If $u$ does not find position, [i.e., $u$ is only "at" $h(1)]$, Figure AC-28, then $r$ will shoot for position. $r$ is shown getting position in Figure AC-31 and failing in Figure AC-32. Rotation forces $k$ to play next. If k's partner ( $u$ ) is in position, then $k$ could try to protect $u$ by shooting to a spot that blocks a shot by $y$ at $u$. An aggressive $k$ could Gamble by advancing half-way to $h(2)$ leaving $u$ unprotected at $h(1)$ while putting $u / k$ in a better position for $h(2)$. After k plays, rotation has y playing next. If he can, y will shoot to clear u . If u is not in position, Figure AC-28, then $k$ should seek position at $h(1)$. $y$ will play next to $h(1)$, and the "real" GC play at $h(1)$ would begin...


In AC-GC: Figure AC-27 and AC-28 are again two possible results of play by the first ball, $u$, in the $h(1)$-Period. And Figures AC-29 $A C-32$ are the possible results for $r / y$ if they play $r$. There is a substantial benefit for $u / k$ in playing $k$, after $u$ and $r$ have played. $k$ satisfies the BBR, gaining the initiative at the Current-Hoop for $u / k$. At that moment, $u / k$ are able to score while $r / y$ cannot. Failure to play $k$ (i.e., playing $u$ again) lets $r / y$ have the initiative at the Current-Hoop because $r / y$ can play their $2^{\text {nd }}$ ball $y$, satisfying the BBR, and be able to score while $u / k$ cannot. Playing $k$ is clearly the path that $u / k$ should follow when $r$ is not in position, as in Figures AC29, $\underline{A C-30}$ and $\underline{A C-32}$. But it need not be the case when $r$ is in position and $u$ is not, as in Figure AC-31. We will discuss $u / k$ 's options in this situation in the section entitled "Return to the Beginning" which follows the next section on (2-on-1)'s.

## The (2-on-1) Battle ${ }^{39}$

The team that plays $2^{\text {nd }}$ in the period of a Current-Hoop faces different issues than the team that plays $1^{\text {st }}-r / y^{\prime} s$ decision to play or not play its $2^{\text {nd }}$ ball ( y ) is different than $\mathrm{u} / \mathrm{k}^{\prime} \mathrm{s}$ decision vis a vis $k$ after $u$ has played. Consider Figures AC-33 - AC-37. These show five different results for the first three balls ( $u, r$, and $k$ ). What should $r / y$ do in their next turn?

Here is the context of the question: If play starts for the Current-Hoop with all four balls around the last hoop or at the Start-in Area, then the team playing $2^{\text {nd }}$ in the Period of the Current-Hoop starts with substantially less than a $50 / 50$ chance of winning the

[^28]Current-Hoop. Our estimate is that even for good players, with $C D=12$, this breaks approximately $85 / 15$ in favor of the team playing $1^{\text {st. }}$. Thus, improving the odds to $50 / 50$ - getting to a ( 2 -on- 2 ) - is a very good result for the $2^{\text {nd }}$ team.

Figure AC-33: No Balls Start in Position: $u$, $r$, or $k$ are not in position at $h(1)$. $y$ should shoot for position at $h(1)$ starting a (2-on-2).

Figures AC-34 and AC-35: $\mathbf{u} / \mathbf{k}$ has one ball in position: There are two main ways this can happen: (i) u plays first to position at $\mathrm{h}(1)$. $r$ shoots at $u$ and misses, and then $k$ shoots but did not get position, Figure $A C-34$. Or (ii) $u$ shoots to position, $r$ shoots at $u$ and hits, clearing u , and then k shoots to position, Figure $\mathrm{AC}-35$. In both of these situtations, $\mathrm{u} / \mathrm{k}$ has only one ball in position.

$r / y$ has two ways it can proceed: (i) Play $y^{40}$ attempting to clear the ball in position. By playing $y, r / y$ satisfies the BBR. If $y$ clears, then there is a (2-on-2). (ii) Play $r$ again. This starts the other major Battle type of AC-GC - a (2-on-1). Here the " 1 " comes without an asterisk because $r / y$ has not met the BBR and therefore $r / y$ cannot score at $h(1)$ until y plays during the $h(1)$-Period. $y$ will get a chance to play (and satisfy the BBR) if $r$ prevails in the (2-on-1). Victory in a (2-on-1) is another path to a (2-on-2).

Should $r$ play again, instead of $y$ ? $r / y$ needs to compare the probability that $r$ wins the ( $2-o n-1$ ) to reach a (2-on-2), with the probability that $y$ clears the ball in position at $h(1)$ and gets to the (2-on-2) directly ${ }^{41}$. Winning a (2-on-1) is biased in favor of the

[^29]2-ball team u/k, (perhaps 67/33) because $u / k$ can score but $r / y$ cannot. $r / y$ may be better off accepting the chance of getting to a (2-on-2) by first fighting and winning a (2-on-1). Evidence from preliminary games of AC-GC suggests that good players are "torn between" attempting to clear from a distance with y or playing the (2-on-1) with r and avoiding the clearance attempt.

Figure AC-36: $\mathbf{u} / \mathbf{k}$ has one ball in position, $\mathbf{r}$ cuddles: $\mathbf{u}$ shoots to position at $h(1)$ and $r$ cuddles - shoots close to $-u$, and then $k$ shoots but did not get position. In this situation, $r$ will have no problem clearing $u$. Compare this to Figure AC-34 where $r$ can clear $u$ but must execute a 7 or 8 -yard come-back clearance shot to do it. Avoiding this shot is the benefit of cuddling. But there is a downside: By not clearing $u$ to begin with, $r / y$ risks the possibility that $k$ joins $u$ giving $u / k$ two balls in position ${ }^{42}$.

Figure AC-37. $u / k$ have two balls in position: $h(1)$ will be won by $u / k$ unless $r / y$ can block both $u$ and $k$ from $h(1)$, as discussed below.

## !!! CLIPS FOR BALLS NOT YET PLAYED TO A HOOP !!!

As a bookkeeping matter, the rules of AC-GC specify that if a Team playing for the $2^{\text {nd }}$ time in a new Period does not play its $2^{\text {nd }}$ ball (i.e., it plays its $1^{\text {st }}$ ball again) then the clip for the $2^{\text {nd }}$ ball is placed on the Current-Hoop and is only removed after that ball has played in the Period, or when that hoop is scored. The clips keep track of which balls have not played during the Period to a hoop.

Defensive Jawsing: One way to block both $u$ and $k$ from scoring when they are both in position, Figure AC-38, is for $r$ to shoot into the Jaws of $h(1)$. Jawsing has defensive and offensive implications in AC-GC because of an exception to the BBR - if $u / k$ causes $r$ or $y$ to score at the Current-Hoop, then $r / y$ wins the Current-Hoop even if one or both of $r$ and $y$ have not played during the Period of the Current-Hoop (i.e., even if $\mathrm{r} / \mathrm{y}$ has not satisfied the BBR). A successful jawsing by r is shown in Figure AC-39. (Note that an attempt to jaws that fails to actually jaws may still be relatively successful if it blocks both Oppos from the hoop). If r runs h(1) without satisfying the BBR then the shot stands but there is no score.

It is $\mathrm{u} / \mathrm{k}$ to play. There are three logical responses:

[^30]1.. Attempt the jump shot: $k$ can attempt to make $h(1)$ with a jump shot, shown successfully completed in Figure AC-40.

2.. Relocate to jump position, or to a better jump position: In Figure $\mathrm{AC}-41$, u has gone to a centered jump position that he prefers. $r / y$ can bring in $y$ or leave $y$ where it is and have $r$ try to defend from his position in the jaws. This can entail one of three efforts: (a) $r$ can try to clear or at least displace $u$ and hope to maybe move $k$ as well, (b) $r$ can move out of the jaws hoping to form an effective block too close to jump, or (c) r can reposition further into the jaws (but not through) so that the slightest of hits during the jump will push $r$ through, scoring the point for $r / y$.
3.. Go to the non-playing side of the Current-Hoop: If $u / k$ does not fancy the jump shot, then $u$ can go to the non-playing side of $h(1)$, Figure AC-42. $u$ is ready to knock $r$ out of the jaws on $u / k$ 's next turn. This will let $u$ take care of $r$, but it also lets y play to the Current-Hoop, starting a (2-on-2). Letting a (2-on-1) become a (2-on-2) due to fear of a jump shot is not a good result for $\mathrm{u} / \mathrm{k}$.

Return to the Beginning: In Figure AC-43, $u$ as T1-B1 does not get position at $h(1)$ but $r$ as T2-B1 does. We explore three possibilities for $u / k$ :
(i) Bring in $k: u / k$ can play $k$ and satisfy the BBR. If $k$ succeeds in getting position at $h(1)$, Figure $A C-44$, then $k$ starts a (2-on-1). If $k$ fails to get position, then $y$ will play, satisfying the BBR for $r / y$. If $y$ gets position Figure $A C-45$, then there is a (2-on-2) with $r / y$ at a significant advantage. Unless $u / k$ can jaws, it is likely that $r / y$ will win at $h(1)$. If $y$ fails to get position then a (2-on-2) starts with both teams in it, Figure AC-46.

(ii) Have u clear r: $u$ can clear $r$, Figure AC-47. If $u$ does not hold position at $h(1)$, then $y$ enters satisfying the BBR for $r / y$. If y finds position, then he starts a ( $1^{*}$-on-1). If y does not get position, then k enters. If k finds position, then he starts a (2-on-1*), both teams are able to score, but $r$ is off in the distance.
(iii) Have u set-up at $\mathbf{h ( 1 )}$ : Finally, if $r$ is in position to make $h(1)$ but $r$ is not also in a good jump position, then $u$ can set-up in front of $h(1)$, blocking $r$ 's path to $h(1)$ and positioning $u$ to jaws on his next shot, Figure AC-48. This creates an interesting problem for $r / y$ : $r$ can clear $u$ (avoiding a rush-peel), but, if $r$ does not remain in position, then $k$ can enter and attempt to find position - if successful then $u / k$ will have the advantage in a ( $2-0 n-1$ ). If $k$ does not find position, then $y$ enters to start a (2-on-2) ${ }^{43}$.

Escaping the Jaws: There is another consideration with jawsing - play after a successful jump by opponents. If the jawsing hoop is odd, then the jawsed ball can likely play to the next even hoop from the jaws. But if it is even, then the jawsed ball must use a shot to get out of the jaws and back into play (which at least helps to satisfy the BBR).

[^31]
## BATtLES INVOLVING BOTH CURRENT AND NEXT-HOOPS

The Battles described so far all took place at or around the Current-Hoop, with little thought of the Next-Hoop. But, AC-GC has no offsides and therefore these Battles can extend to Next-Hoops. There are two reasons this can be of value: (i) To help secure the Next-Hoop when losing at the Current-Hoop is a foregone conclusion, and (ii) To Gamble for a chance at two hoops in-a-row.

Helping to Secure the Next-Hoop: In Figure AC-49, u opens the h(1)-Period shooting to position at $h(1)$. $r$ shoots at $u$ and misses. k lags to position. Both $u$ and $k$ have easy hoop shots. $u / k$ should win at $h(1)$. It is $r / y$ to play, what should they do? r/y should "presend" y to $\mathrm{h}(2)$, with two possible results: Figure $\mathrm{AC}-50$, y finds position at $\mathrm{h}(2)$, or, Figure $\mathrm{AC}-51$, y is "at" $\mathrm{h}(2)$ but not in position.


We assume that competent GC players can send a ball from anywhere on the lawn to position at any hoop virtually all of the time with two shots and that a ball already "at" a hoop, but not in position, goes to position without difficulty on its next shot. But the extra shot is needed. Thus, y in position, Figure $\mathrm{AC}-50$, is significantly more valuable to $\mathrm{r} / \mathrm{y}$ than y only "at" the hoop, Figure $\mathrm{AC}-51$.

We assume $u / k$ plays $u$ and scores at $h(1)$ after $y$ is pre-sent to $h(2)$, Figure $A C-52$. (This need not be the case as is discussed below). The $\mathrm{h}(1)$-Period is over and the game enters the $\mathrm{h}(2)$-Period.

The team that lost at $h(1), r / y$, always plays first in a new Period.

As noted, playing first in the Period of the Current-Hoop is valuable because it allows $r / y$ to satisfy the BBR before $u / k$ has a chance to do so, putting $r / y$ on offense and $u / k$ on defense. But, being first in a Period is even more valuable if $r / y$ gets one, or both, of $r$ and $y$ to position at the now Current-Hoop, h(2), with their first shots after $u$ has scored the last hoop, h(1). Figures AC-53, AC-54 and AC-55 show three possible outcomes when $r$ plays first for $r / y$. In each case, only $r$ has played in the $h(2)$-Period.

Figure AC-53: Both $r$ and $y$ are in position. It is $u / k$ to play. $u / k$ can shoot to clear $y$. If this happens, then $y$ should shoot back to position satisfying the BBR. With two balls in position this becomes a replay of figure AC-49 from $h(1)$, now played out at $h(2)$. $\mathrm{r} / \mathrm{y}$ should win at h(2). (As an aside, a successful Hail-Mary shot by $u$ or $k$ that Jawses at $h(2)$ would delay progress by $r / y$ ).

Figure AC-54: $r$ is in position, $y$ is not. $u$ (or $k$ ) should shoot close to $h(2)$, $y$ will shoot to position (and get it), satisfying the BBR. $r / y$ has two balls in position at $h(2)$. u's only hope is to jaws. r/y should win at $h(2)$.

Figure AC-55: Neither r nor y are in position. $u$ (or $k$ ) should shoot near $y$, north and west of the hoop. $r$ has already played. y plays next, satisfies the BBR, and finds position. $u$ clears $y$ and starts a (2-on-1). $r / y$ has the advantage.

Conclusion: Pre-sending a ball to the Next-Hoop shields the pre-sending team from an immediate (2-on-2) at that hoop and puts them in a dominant position when the Next-Hoop becomes the Current-Hoop - with the possibility of a Quick Score or a (2-on-1) where they are the 2-ball team.

Offensive Jawsing at Odd-Numbered Hoops: In Figure AC-52, y was pre-sent to $h(2)$ and then u made $h(1)$ ending the $h(1)$-Period. But u had another option - to defer making h(1) by jawsing, Figure AC-56. Jawsing an odd-numbered hoop allows $u$ to proceed to the next even-numbered hoop in the stroke that is used to make the jawsed hoop. Here u can make $h(1)$ and simultaneously proceed to $\mathrm{h}(2)$ as a defensive ball to contend with the pre-sent ball, y . It is $\mathrm{r} / \mathrm{y}$ to play next. We consider three options:
(i) $\quad r$ peels $u$ : A good response is for $r$ to peel $u$, Figure AC-57. This gives up the point at $h(1)$ but should leave $u$ in a position that is no better, and could be worse, than the one $u$ would have had if $u$ made $h(1)$ on his own instead of jawsing. Ideally the peel would leave $u$ with a hampered shot limiting $u$ 's ability to get to $h(2)$ from $h(1)$. But getting the peel done is paramount!
$u$ loses the ability to proceed to $h(2)$ as he makes the $h(1)^{44}$. The benefits to $u / k$ of $u$ jawsing have been negated - the benefits to $r / y$ of pre-sending $y$ to $h(2)$ are maintained ${ }^{45}$.
(ii) $y$ goes to position: If $y$ is out of position, as in Figure AC-56, then $y$ can shoot to position as shown in Figure AC-58. Next, $u$ makes $h(1)$ ending the $h(1)$-Period, and simultaneously proceeding to $h(2)^{46}$. Then $r$ plays starting the $h(2)$-Period, Figure AC-59. If $r$ gets position, then $r / y$ should win at $h(2)$, jumping with $y$ if $u$ jawses. If $r$ does not get position, then $u / k$ can try for a (2-on-2) by having $k$ clear y from afar [back at $h(1)]$, or $u$ can start a (2-on-1) by clearing $y$.
(iii) $r$ goes to the Next-Hoop: $r$ could go to $h(2)$ in anticipation of $u$ preceding to $h(2)$ as $u$ scores $h(1)$. The trouble with this strategy is that $u$ can stay jawsed and send $k$ instead, Figure AC-60. u can "hang out" in the jaws of $h(1)$ and only proceed to $h(2)$ later, hoping to create a (2-on-2) which would be a great result for $u / k$.


[^32]Offensive Jawsing at Even-Numbered Hoops: Figure AC-61: The Current-Hoop is $\mathrm{h}(2)$, an even-numbered hoop. Both u and k are in position. Figure AC-62: r/y gives up on $h(2)$ and pre-sends $y$ to $h(3)$. y may or may not get position. u jawses $h(2)$, Figure AC-63.

All that $u$ can do from the jaws of an even-numbered hoop, $h(2)$, is to make the hoop! $u$ cannot also proceed to the next now oddnumbered hoop, $\mathrm{h}(3)$. This illustrates the fundamental difference between the odd and even hoops as the Current-Hoop when Striker considers jawsing. Once u jawses at an even hoop r/y have three relevant choices:
(i) $\quad r$ can join $y$ at $h(3)$ completely giving up on $h(2)$, Figure $A C-64$. But this would allow $u / k$ to send $k$ to $h(3)$, Figure $A C-65$. $u$ will wait to score until $k$ succeeds in driving one or both of $r$ and $y$ out of position or $k$ finds an opportunity to jaws. This is not a desirable result for $r / y$.
(ii) $\quad r$ peels $u$ at $h(2)$. Then $r$ plays again shooting to $h(3)$ to start the $h(3)$-period. The peel ensures that $r / y$ will have two balls at $h(3)$, Figure AC-66.
(iii) $\quad r$ goes to the non-playing side of $h(2)$, Figure AC-67. From here, $r$ threatens to rush $u$ out of $h(2)$. $u$ is forced to make $h(2)$. $r / y$ play first in the $h(3)$ period allowing $r / y$ to have two balls at $h(3)$ before $u / k$ plays.


## GAMBLING

The "natural" path to victory in an AC-GC game is to win every time your team goes first to a hoop (i.e., every time it goes first in a new Period) and then to capitalize on the mistakes of your opponents to win at least one more time when they go first than they win when you go first. This is a challenge because winning at the Current-Hoop is heavily biased in favor of the the team playing first to it. It often means starting at a disadvantage and progressing to a (2-on-2), either with a long clearance shot at a ball in position or winning a (2-on-1) starting as the 1-ball team, and then winning that now $50 / 50$ Battle by scoring the hoop.

But there is another path to victory - by Gambling. This is where a team takes action to increase its probability ("Prob") of winning the Current-Hoop (CH) and the Next-Hoop (NH). A team Gambles by intentionally weaking its position at the CH, in order to strengthen it at the NH. This lowers the Prob[win at CH ], while increasing the Prob[win at NH]. But more importantly, it increases what matters most, the product of the probabiities - $\operatorname{Prob}[w i n C H]^{*} \operatorname{Prob}[w i n \mathrm{NH}]$ - that is, the probability of winning at both hoops Prob[win CH and NH]. We identify two forms of Gambling - what we call the Mini and Maxi-Gambles and present them by example below, involving $h(2)$ as the CH and $\mathrm{h}(3)$ as the NH .

The Mini Gamble: Suppose that during the $h(1)$-Period $u / k$ takes control of the CH, $h(1)$. $r / y$ gives up on $h(1)$, pre-sends $y$ to the NH, $h(2)$, and $y$ finds position at $h(2)$. u then makes $h(1)$, ending the $h(1)$-Period. Figure AC-68. Now $r / y$ plays first in the $h(2)$-Period. This is the decision point for a Mini-Gamble: $r / y$ can decline the Mini and send $r$ as T1-B1 to $h(2)$, Figure AC-69, or r/y can take the Mini and send $r$ to $h(3)$, Figure $A C-70$ where $r$ may or may not find position. These possibilities are shown by having two red balls at $h(3)$.


The Maxi-Gamble: Suppose u makes $h(1)$ when all four balls are in the vicinity of $h(1)$. This ends the $h(1)$-Period, Figure AC-71. y plays first in the $h(2)$-Period as T1-B1. y shoots to $h(2)$ and finds positition, Figure AC-72. k plays next, as T2-B1, and cuddles up to $y$, Figure AC-73. It is $r / y$ to play. This is the decision point for the Maxi: $r / y$ declines the Maxi if $r$, as T1-B2, shoots to h(2), Figure AC-74. $r / y$ accepts the Maxi-Gamble if $r$ shoots to $h(3)$, Figure AC-75. No matter what $r$ chooses to do, it may or may not find position, again shown with two red balls at $\mathrm{h}(3)$.


Comparing the Mini and Maxi Gambles: $r / y$ is in a much stronger position at $h(2)$ after taking the Mini than after taking the Maxi. This can be seen by comparing the decision point of the Mini, Figure AC-68, to the decision point of the Maxi, Figure AC-73. In both cases, play has entered the $h(2)$-Period with $y$ in position at $h(2)$. And, in both cases, it is $r / y$ to play. But there is a major difference: in the Mini, $r$ will play as T1-B1 - $y$ was pre-sent to $h(2)$ during the $h(1)$-Period. In the Maxi, $r$ will play as T1-B2 - $y$ was sent to $h(2)$ to start the $\mathrm{h}(2)$-Period as T1-B1 and was followed to $\mathrm{h}(2)$ by an opponent (k) as T2-B1. If $\mathrm{r} / \mathrm{y}$ takes the Mini, Figure $\mathrm{AC}-70$, then $\mathrm{u} / \mathrm{k}$ has one long low-probability clearance shot at $y$. But if $r / y$ takes the Maxi, Figure $A C-75$, then $k$ has a high-percentage clearance shot.

Valuation Procedure: We valued the Mini and Maxi Gambles using a decision tree approach that traces through relevant plays that can happen in the scoring of the Current-Hoop and the Next-Hoop. We did the calculations with a player having a CD=12 in mind. This led to the following assumptions for relevant variables ${ }^{47}$ :

[^33]
## Variables and Values

| $(2$-on-2) | is won by either team | $=.5$ |
| :--- | :--- | :--- |
| $\left(1^{*}\right.$-on-1) | is won by the $1^{*}$-ball team | $=.55$ |
| $(2$-on-1) | is won by 1-ball team | $=1 / 3^{48}$ |
| primary clearance |  | $=1 / 3$ |
| ball shoots to position | $=2 / 3$ |  |

Calculations are made easier by the fact that, if $r / y$ wins $h(2)$, then the probability that $r / y$ wins $h(3)$ is a function of whether a Gamble was taken, or not, but it is independent of which Gamble was taken. Figure AC-76 shows the state of play if $r / y$ wins at $h(2)$ but $r / y$ did not take either Gamble. Here, $y$ is shown scoring at $h(2)$ and, since a Gamble was not taken, $r$ was sent to $h(2)$ and will be around $\mathrm{h}(2)$ when $\mathrm{h}(2)$ is made. (It is also possible that $r$ wins the hoop with $y$ remaining nearby). Figures AC-77 and AC-78 show the state of play if $y$ wins $h(2)$ and $r / y$ took either Gamble. In this case, $r$ was pre-sent to $h(3)$ instead of engaging at $h(2)$. Two possible outcomes are shown because $r$ may just be "at" $h(3)$, or $r$ may have achieved position at $h(3)$. The probabiliy of winning at $h(3)$ conditional on $r$ taking a Gamble will be the results achieved from Figures AC-77 and AC-78, weighted by the probability that $r$ is "at" $h(3)$ or in position at $h(3)$ - that is, by the probability that $r$ shot to position at $h(3)$ when $r$ initiated the Gamble.

Ball Positions if $r / y$ wins $h(2)$


[^34]
## Conclusion - Gambling should be an important part of the game of AC-GC!

In the Appendix (found at the end of this chapter): Calculating the Probability of Winning Two Hoops in a Row, we use these assumptions to calculate the Prob (win CH and win NH) with: (i) no Gamble - as $21 \%$, (ii) the Mini Gamble - as $47.7 \%$, and (iii) the Maxi-Gamble - as $42.2 \%$. Thus, the Gambles more than double the probability of winning two hoops in a row! (While not specifically analyzed in the appendix, Gambles are still worthwhile under a wide range of relevant values for the underlying variables) ${ }^{49}$.

## Terminating or Morphing Gambles



Figure AC-79 reprises the start of a Maxi-Gamble with two r's at h(3), one in position and one not. In Figure AC-80, the Gamble plays out with k clearing y , starting a (1-on-1*) in $\mathrm{r} / \mathrm{y}^{\prime} \mathrm{s}$ favor, with u pinned at $\mathrm{h}(1)$. y needs to get back to position at $\mathrm{h}(2)$ in order to continue this Battle. However, y fails to get back to position, Figure AC-81. This allows $u / k$ to bring $u$ from $h(1)$ to participate in this h(2)-Battle, Figure AC-82.

What $\mathrm{r} / \mathrm{y}$ does next is dependent on r 's position at $\mathrm{h}(3)$ and u and k 's positions at $\mathrm{h}(2)$.

[^35]If $r$ is in position at $h(3)$, then $y$ should play. What $y$ does depends on where $u$ and $k$ are. If only one of them is in position at $h(2)$, then $y$ should clear it. If neither $u$ nor $k$ are in position, then $y$ should shoot back to position, Figure AC-83. In either case, $y$ will fight the now ( $2-$ on- $1^{*}$ ) as the $1^{*}$-team with less than a $50 / 50$ chance of winning at $h(2)$. If both $u$ and $k$ are in position, then $r / y$ are destined to lose at $h(2)$. $y$ should force $u / k$ to score by staying around $h(2)$ to prevent a jawsing. Once $u / k$ scores $h(2), y$ can immediately shoot to $\mathrm{h}(4)$, as T1-B1 in the $\mathrm{h}(3)$-Period, starting a Mini-Gamble.

If $r$ is out of position at $h(3)$, and neither $u$ nor $k$ are in position, then $r$ can return from $h(3)$ to $h(2)$, starting a (2-on-2). This would terminate the Maxi-Gamble but give $r / y$ a $50 / 50$ chance at $h(2)$, Figure AC-84. After y fails to return to position at $h(2)$, if $u$ finds position at $h(2)$, then $r$ has a choice. He can clear $u$ and fight a $\left(2-o n-1^{*}\right)$ as the $1^{*}$-team with less than a $50 / 50$ chance of winning, or $r$ can go to position at $h(3)$, Figure AC-85. If, as expected, $u$ then scores $h(2)$, Figure AC-86, then $y$ can shoot to $h(4)$, as the first ball to play in the $h(3)$-Period, knowing that $r$ is in position at $h(3)$. This strategy locks in a loss at $h(2)$ where $r / y$ had less than a 50/50 chance of winning, but allows $r / y$ to start a Mini-Gamble where they should win $h(3)$ - the Current-Hoop - easily, and have a good chance at $\mathrm{h}(4)$ - the Next-Hoop, Figure AC-87.


AC-83
AC-84


AC-86

The point of this example is that starting a Gamble does not lock a team into carrying it through to the end. And, separately, failure of one Gamble can lead to another ${ }^{50}$.

[^36]
## Responding to a Mini-Gamble

Consider Figure AC-88. u won $h(1)$ for $u / k$ to end the $h(1)$-Period. But before the period ended, $r / y$ gave up on $h(1)$ and pre-sent $y$ to $h(2)$. As the loser at $h(1)$, $r / y$ played first to start the $h(2)$-Period by sending $r$, as T1-B1, to $h(3)$, establishing a Mini-Gamble, hoping to win $h(2)$ and $h(3)$. It is $u / k$ to play a ball as T2-B1. What should $u / k$ do? $u / k^{\prime} s$ decision should consider carefully $r^{\prime} s$ position at $h(3)$ !

| $r$ in Position at $\mathrm{NH}=\mathrm{h}(3)$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $u / k$ contests $\mathrm{h}(2)$ |  |  |  |  | $\mathrm{u} / \mathrm{k}$ concedes $\mathrm{h}(2)$ |  |
| Mini started | $k$ misses y | y makes $\mathrm{h}(2)$ | k misses r | y to $\mathrm{h}(3)$ | $k$ to h(3) | double play by $k$ |
| $\bigcirc$ | $\bigcirc$ | $\stackrel{\circ}{i}$ | \% | $\square \quad$ i | $\stackrel{\bullet}{\square}$ | \% |
| - |  |  | - $\pi$ | - $\pi$ | - $\pi$ | - $\pi$ |
| AC-88 h(2) | AC-89 h(2) | AC-90 h(3) | AC-91 h(3) | AC-92 h(3) | AC-93 h(3) | AC-94 h(3) |

(i) If $\mathbf{r}$ is in position at $\mathrm{h}(3): \mathrm{u} / \mathrm{k}$ has two basic options: to compete at $\mathrm{h}(2)$, or to concede that hoop.
a.. If $u / k$ chooses to compete at $h(2)$, then the odds suggest that $u / k$ is likely to lose both $h(2)$ and $h(3)$ : $k$ tries to clear $y$ and misses, Figure AC-89; y makes $h(2)$ to end the $h(2)$-Period, Figure AC-90; As the losers at $h(2), u / k$ starts the $h(3)-$ Period by having $k$ try to clear $r$ at $h(3)$. This fails, Figure AC-91; y shoots to $h(3)$ as T2-B1, Figure AC-92. If y finds position, then, with both balls in position, $r / y$ should win at $h(3)$. If $y$ does not find position, then $r / y$ will still control $h(3)$.
b.. $u / k$ can concede $h(2)$ and send a ball (k) to $h(3)$ as T2-B1 in the $h(2)$-Period to defend against $r$, Figure AC-93. After y makes $h(2)$ to end the $h(2)$-Period, $u / k$ can play $k$ again, as T1-B1 to $h(3)$, starting the $h(3)$-Period, clearing $r$, and also
gaining position at $h(3)$, Figure AC-94. This "double play", clearing $r$ and gaining position at $h(3)$ at the same time, is possible if $k$ was carefully placed at $h(3)$ and then $k$ does not try to clear $r$ too far away ${ }^{51}$.
(ii) If $r$ is not in position at $h(3): u / k$ still run the risk of losing two hoops in a row if they contend at $h(2)$. If $u / k$ concedes $h(2)$ then $u / k$ can get one ball to $h(3)$ during the $h(2)$-Period and follow it with the other ball to start the $h(3)$-Period, thus controlling $\mathrm{h}(3)$.

## Responding to a Maxi Gamble

Figure AC-95: $u$ (T1-B1) starts the $h(1)$-Period by shooting to position at $h(1), r$ (T2-B1) lags to $h(1)$ (or shoots at $u$ and misses), and then $k$ (T1-B2) lags to $h(2)$ and not $h(1)$. This is the defining position of the Maxi-Gamble. $u$ and $k$ have both played in the $h(1)$-Period for $u / k$ (T1), satisfying the BBR, but only $r$ has played for $r / y(T 2)$. y (T2-B2) is waiting in the wings on the Start-in Baulk-Line. The stage is set for a ( $1^{*}$-on-1) between $u$ and $r$ at $h(1)$, with $u$, but not $r$, having the ability to score ${ }^{52}$.

In Figure AC-96, We show $u$ scoring $h(1)$ to win the ( $1^{*}-\mathrm{on}-1$ ) and end the $\mathrm{h}(1)$-Period. Attention turns to $\mathrm{h}(2)$ with $\mathrm{r} / \mathrm{y}$ playing first as T1 to $h(2)$ to start the $h(2)$-Period. Except for the possibiliy of a $2^{\text {nd }}$ Shot Quick Score - a very long hoop shot by r or y - with k already in the neighborhood, $k$ can virtually guarantee that $h(2)$ turns into at least a ( $2-o n-1$ ) with $u / k$ as the 1-ball team. And it might even become a (2-on-2) if $k$ is able to drive T1-B1 (whichever ball $r / y$ sends first) a long way from $h(2)$, and then neither $r$ nor $y$ are able to return to position. This would be a very good outcome for $u / k$ - winning at $h(1)$, and having a real chance to win at $h(2)$.

In Figure AC-97, We show the ( $1^{*}$-on-1) a few turns further on, this time assuming that: (i) r drives $u$ out of position, (ii) $u$ fails to return to position, and (iii) y enters the fray by shooting to $h(1)$. u could begin a (2-on-2) that would be played out (2-on-1*), with

[^37]$k$ dangling at $h(2)$. In Figure $A C-98, ~ u / k$ ignore the positions of $r$ and $y$ (even if $y$ is in position!), and instead perfect the position of $k$ at $h(2)^{53}$ ! If $y$ or $r$ are in position at $h(1)$, then this will allow $r / y$ to make $h(1)$ as shown in Figure AC-99. But, with $k$ in good position at $h(2), \mathrm{u} / \mathrm{k}$ could respond by sending u to $\mathrm{h}(3)$ (not $\mathrm{h}(2)$ !) to start the $\mathrm{h}(2)$-Period, from the position shown in Figure AC-99. In this case, $u / k$ loses $h(1)$, but has a high probability of winning $h(2)$, and is positioned to create mischief at $h(3)$ by starting a Mini-Gamble, Figure AC-100.


However, $r / y$ have another play. If $u$ fails to return to position at $h(1)$ during the ( $\left.1^{*}-o n-1\right), r / y$ can send $y$ to $h(2)$ instead of $h(1)$ ! This would trade a (2-on-1*) at $h(1)$, where $r / y$ has an advantage, for a ( $1^{*}$-on- $1^{*}$ ) at $h(1)$, where it should be $50 / 50$, but it would limit the advantage that $u / k$ has at $h(2)$, as an unopposed ball, by setting up a "pre" (1-on-1) at $h(2)$ involving $y$ and $k . u / k$ could counter by sending $k$ back to $h(1)$. If $k$ does this and gets position at $h(1)$, then there would be a ( 2 -on-1*) at $h(1)$. If $k$ fails to get position at $h(1)$, then $y$ could return to $h(1)$ as well, and there would be a full-fledged (2-on-2) at $h(1)$.

[^38]
## APPENDIX: CALCULATING THE PROBABILITY OF WINNING TWO HOOPS IN A ROW

NO GAMBLE: If all balls start around the just scored last hoop, as is shown in Figure AC-71, and no Gamble is in place and none will be taken, then the team playing $1^{\text {st }}$ to the CH has a $70 \%$ chance of winning the hoop. Since one team or the other will win the hoop, this means that the team that plays $2^{\text {nd }}$ has a $30 \%$ chance. Treating AC-GC as a series of repeating single-hoop games means that these same probabilities apply to the NH and all future hoops. Therefore, the chance of winning two hoops in a row, in the absence of Gambles, is the product of the individual probabilities ( 0.7 * 0.3)- hence $21 \%$.

MINI IN PLACE: Suppose the balls are positioned for a possible Mini-Gamble, Figure AC-68. y was pre-sent to h(2) and found position. However the Gamble may or may not be taken with $u$ having just scored at $h(1)$ and $r / y$ to play. Clearly, $r / y$ are in a stronger position with respect to making $h(2)$ here than they were back in Figure AC-71, with y around $h(1)$.

If the Mini is not taken, then $r / y$ has a $98.5 \%$ chance of winning at $h(2)$, getting to Figure AC-69 from Figure AC-68, which is $28.5 \%$ higher than it is if $y$ is not in position at $h(2)-[98.5 \%$ versus 70\%]. If no Gamble is organized for $h(3)$, then $r / y$ 's odds for making the NH are unchanged from what they are in the No Gamble situation $-r / y$ will have a $30 \%$ chance of winning h(3) and a ( $0.985 * 0.3$ ) $=29.6 \%$ chance of winning two hoops in a row. The chance of winning both hoops has increased by $8.6 \%$, [29.6\% versus $21 \%$ ].

If the Mini is taken, then r/y has an $84.9 \%$ chance of winning at $h(2)$. This is higher than the no Gamble case, but less than the result if the Mini is not taken. But now, because the Mini is taken, the chance that $r / y$ wins at $h(3)$ increases to $56.2 \%$ and the probability of winning at both $h(2)$ and $\mathrm{h}(3)$ becomes $(0.849$ * 0.562$)=47.71 \%$ - more than double the no Gamble case ( $21 \%$ ).

MAXI IN PLACE: Suppose the balls are positioned for a possible Maxi-Gamble, Figure AC-73. y found position at $h(2)$ to start the $h(2)$-Period and k followed. However the Gamble may or may not be taken.

If the Maxi is not taken, then $\mathrm{r} / \mathrm{y}$ has a $94.4 \%$ chance of winning at $\mathrm{h}(2)$, getting to Figure AC-76 from Figure AC-74, which is $4.1 \%$ lower than it is in the case when the Mini is not taken - [94.4\%-98.5\%] but still $24.4 \%$ higher than the no Gamble case [94.4\% - $70 \%$ ]. The chance of winning two hoops in a row is $\left(.944^{*} 0.3\right)=28.3 \%$. As expected, this is higher than the no Gamble case, $(28.3 \%$ vs $21 \%)$ but lower than the case for the Mini.

If the Maxi is taken, then r/y has a $75 \%$ chance of winning at $h(2)$. This is $5 \%$ higher than the no Gamble case (70\%) but, $9.9 \%$ less than the Mini case [84.9\% vs 75\%]. Finally, the probability of winning two hoops in a row becomes $42.2 \%$ ( 0.75 * 0.562 ), once again, double the no Gamble case.

## Data from the Videos

Videos from five AC-GC games are available. The first three involved the Fearsome Foursome playing doubles. The final two involved the Twosome (Matthew and Stephen) playing singles.

The original games resulted in scores scores of (5-4), (5-4), and (5-3), respectively, thus, close games involving a total of 26 points. The games took 34,19 and 26 minutes, an average of just over 26 minutes. Here is an analysis of how the games were played. That is we coount and discuss Quick-Scores, and various battles, notably the (2-on-1) and the (2-on-2) that occurred.

Of the 26 points played:
18 First Shots Achieved Position: to start off the period of a new hoop. This is approximaely $70 \%$ which is consistent with the percentage of hoops the Fearsome Foursome made during the 1-Ball test - 73\%.

2 were Quick-Scores: Involving only 3 shots during the period of the Current-Hoop. Both occurring after a team had abandoned the previous hoop to pre-send a ball ahead to the next one.

11 started as (2-on-1)s: With only 3 balls initially contesting at the Current-Hoop. 5 were won outright as (2-on-1) and 6 progressed from ( $2-$ on-1) to ( 2 -on-2). Of these 6 , the team initially with two balls in the ( $2-1$ ) won the ( $2-0 n-2$ ) 3 times $-1 / 2$ of the time.

13 progressed to (2-on-2)s: With 4 balls immediately played during the period of, and to the Current-Hoop. Of these 10,7 were won by the team that played $1^{\text {st }}$ to the hoop and 3 were won by the team that played $2^{\text {nd }}-$ when they "broke" serve.

7 involved Jawsing: 4 were at odd hoops where Jawsing is powerful, and 3 were at even hoops where it does not add value.
5 serves were broken: Only once in the $1^{\text {st }}$ and $3^{\text {rd }}$ games, and 3 times in the $2^{\text {nd }}$.
5 involved going to the next hoop: Giving up on the Current-Hoop.
Gambling: We were disappointed - No Mini or Maxi Gambles were taken in these first three games! We attributed this to the fact that we had not fully worked out or explained the "economics" of gambling at that time. Since then we have spent time discussing

Gambling with the players, first with David Maloof and Jeff Soo as a precursor to their demonstrations (not videoed) in north Carolina in June 2021 and then with the Twosome - Matthew and Stephen for the second set of videos included herein. While not an obvious play to the uninitiated (but then, what about croquet is?), Gambling is more and more valued and used as participants try it out and become familiar with it. It is the defining force of AC-GC.

Unfortunately I was not as precise as I wanted to be in my identification and discussion of Gambling as I announced these games. Here is what you need to remember:

First what Gambling is not: While identified as "a form of Gambling" in the video, leaving a fight at the Current-Hoop that is hopeless to get ahead at the Next-Hoop is not Gambling! It is smartly using the fact that this AC-GC has no offsides, but it is definitely not Gambling! Sorry for this confusion.

What Gambling is: The possibility of taking a Gamble occurs when your team plays as T1 - the first team to the Current-Hoop. It is having or placing a ball in position at the Current-Hoop and then deciding what to do with your other ball. If that ball is sent to the Next-Hoop instead of to the Current-Hoop, then you are Gambling! A Mini-Gamble is where the first ball you play, T1-B1, is sent to the Next-Hoop because the ball you plan to play second, as T1-B2, is already at the Current-Hoop due to play at the Previous-Hoop. A Maxi-Gamble is where T1-B1 is sent to position, T2-B1 plays and then T1-B2 is sent to the Next-Hoop.

Game Descriptions: In what follows, the result can either be "NORMAL" meaning that T1 won, or it can "BREAK" meaning that T2 won. Where appropriate, the result can be identified as being part of a "GAMBLE". Finally we provide an on-going tally of the score.

## Game \#1 - Matthew vs Stephen

We have stressed that having T1 win is "NORMAL" and that "BREAKs" are hard to come by. This game is an exception where of the eight hoops played, five were won with BREAKs. And, delightfully, there was a important GAMBLE!
h(1): In AC-GC the team playing first to a hoop, T1, has an advantage because he can get both balls played to the Current-Hoop (T1-B1 and T1-B2) while the $2^{\text {nd }}$ team has only played its $1^{\text {st }}$ ball, T2-B1. But, for this to have value for T1, at least one of its balls needs to gain position on these initial shots. In this case Matthew, as $u / k$, was T1 but did not get either $u$ or $k$ to position at $h(1)$. This allowed Stepen to bring his $2^{\text {nd }}$ ball from the Starting-Area to $h(1)$, causing a (2-on-2) Battle that Stephen eventually - "breaking Matthew's Serve" and scoring the point for $r / y .(u / k: 0, r / y: 1)$ - BREAK for $r / y$.
$\mathbf{h ( 2 ) : ~ T h e ~ t e a m ~ ( h e r e ~} u / k$ ) that loses at the Prior-Hoop always goes first to the Next-Hoop and has the advantage as $T 1$ but needs to get position to benefit. Matthew played first and won this hoop. (u/k:1, r/y:1) - NORMAL for $u / k$.
$\mathbf{h}(3)$ : r/y played first and won the hoop. $r$ scored $h(3)$ from the jaws of this odd hoop making his way down to position at $h(4)$. An excellent result that put tremendous pressure on $u / k$. (u/k:1, r/y:2) - NORMAL for $\mathrm{r} / \mathrm{y}$.
$\mathbf{h ( 4 ) : ~} \mathrm{u} / \mathrm{k}$ was T 1 to this hoop but Matthew failed to get position with T1-B1. Steve got position with T2-B1 and now had two balls in position. Matthew cleared one of them hoping for a miracle. Stephen scored! (u/k:1, r/y:3) - BREAK for $r / y$.
$\mathbf{h ( 5 ) : ~ M a t t h e w ~ i s ~ n o w ~ d o w n ~ t w o ~ b r e a k s ! ~ H e ~ s t a r t s ~} h(5)$ by sending $u$ to position. $r$ shoots and misses. Then Matthew GAMBLES $-a$ Maxi-Gamble at a middle hoop - by shooting $k$ to $h(6)$ while we are still in the period of $h(5)$. Back at $\mathrm{h}(5) \mathrm{y}$ misses and Matthew scores with $u$. (u/k:2, r/y:3) - GAMBLE for $u / k$.
$\mathbf{h ( 6 ) : ~ W h e n ~ y o u ~ p r e - s e n d ~ a ~ b a l l ~ t o ~ t h e ~ N e x t - H o o p , ~ a s ~ M a t t h e w ~ d i d ~ w i t h ~} k$ when we were still in the Period of $h(5)$, your minimum hope is to get both balls in play and fight a (2-on-2) Battle and hopefully prevail54. This is what happened at $h(6)$. There was an extended (2-on-2) Battle with Matthew prevailing. (u/k:3, r/y:3) - BREAK for $\mathbf{u} / \mathbf{k}$. Note carefully - even though the score was now tied, $r / y$ was still up a break as they would go first to $h(7)$.
$\mathbf{h}(7): r / y$ played first as planned but succumbed to a (2-on-2). Part of the reason this happened is because of the proximity of $h(7)$ to $h(6)$ which made clearance shots easier. (u/k:4, $\mathrm{r} / \mathrm{y}: 3$ ) - BREAK for $\mathbf{u} / \mathrm{k}$. Each team has inflicted and endured 2 BREAKs. The game is essentially tied. u/k can win 5-3 by breaking, or we can go into over time. r/y will play first to $h(8)$. However there is one important thing on $u / k$ 's side: $u$ jawsed at this odd-numbered hoop and scored while gaining position at h(8)!
$\mathbf{h ( 8 ) : ~ r / y ~ p l a y e d ~ f i r s t ~ w i t h ~ y ~ a s ~ T 1 - B 1 , ~ a t t e m t i n g ~ t o ~ c l e a r ~ u . ~ y ~ m i s s e d ~ a n d ~} k$ shot to position as T2-B1. R missed as T1-B2 and u scored as T2-B2 to win the game. ( $u / k: 5, r / y: 3$ ) - BREAK for $u / k$ and the WIN for $u / k . r / y$ was in a tough spot once $u$ scored $h(7)$ and also found position at $h(8)$. Stephen played this hoop as he might play a hoop in $G C$ - shooting at $u$. But an alternative would have been for $y$ (or r) to shoot to position. If $k$ does not find position then y could fight a desperate (2-on-1)...

[^39]
## Game \#2 - Matthew vs Stephen

This time Stephen played $u / k$ and went first, as is required of $u / k$ in order to accomodate the scoring system that has $u / k$ win by 2 while $r / y$ can win by 1 , as the winner acumulates at least 5 points. Matthew played second with $r$. Once again, there were a surprising number of BREAKs and one GAMBLE. One interesting thing to watch was Matthew promoting one of his balls (his $2^{\text {nd }}$ in the period) with the other (his $1^{\text {st }}$ in the period). If the promotion gets the $2^{\text {nd }}$ ball to position, then it can score with its first shot.
$\mathbf{h ( 1 ) : ~} u$ shot to position, $r$ cleared to the west boundary, $k$ shot short and did not find position so $y$ came in. A (2-on-2) ensued, that $y$ eventually won for $\mathrm{r} / \mathrm{y}$ with a very good, short, angled hoop shot. ( $\mathbf{u} / \mathrm{k}: \mathbf{0}, \mathrm{r} / \mathrm{y}: 1$ ) - BREAK for $\mathrm{r} / \mathrm{y}$. It can be very costly for 71 if neither of his balls get position with their initial shots. Such failure guarantees a (2-on-2) which is often a 50/50 proposition.
$\mathbf{h ( 2 )}$ : $u$ shot to position; $y$ went near to $u ; k$ shot to position. $u / k$ had two balls in position so $r$ gave up and went to $h(3)^{55} . u / k$ choose not to score immediately but destroyed y instead. y went to $h(4)$ hoping to start a Gamble at $h(3)$. $u / k$ sent $k$ to $h(3)$ to battle with $r$ while $u$ held the fort at $h(2)$. A "pre-battle" ensued at $h(3)$ while $h(2)$ was still unresolved. Eventually $u$ scored at $h(2)$, with $r$ and $k$ still at $h(3)$, and $y$ near $h(2)$ after $y$ shot at $u$ and missed from $h(4)$. ( $u / k: 1, r / y: 1)$ - NORMAL for $u / k$.
$\mathbf{h ( 3 )}$ : y shot to position; k cleared the pre-sent ball r and jawsed in the process. r came back leaving both r and y in jump positions. u still needed to play and went to $h(4)$ assuming $r$ or $y$ would jump and win. However, $r$ missed the jump-shot letting $k$ score from the jaws of $h(3)$, an odd-numbered hoop and progress down the next hoop, $h(4)$. The missed jump by $r / y$ was a costly mistake. (u/k:2, $\mathrm{r} / \mathrm{y}: 1)$ - BREAK for $\mathrm{u} / \mathrm{k}$. This was a lucky break for $\mathrm{u} / \mathrm{k}$.
$h(4)$ : Although $r / y$ is T1, $u / k$ started this hoop with both balls at $h(4)$. y promoted $r$ from $h(3)$ down toward $h(4)$ - a long hoop shot in an effort to stay in contention for the hoop. $u$ shot to position and then $r$ tried the hoop and failed. $k$ shot to position giving $u / k$ two balls at $h(4)$ in position. $r / y$ gave up and had $y$ shoot to $h(5)$ where he found position. $u$ scores ( $u / k: 3, r / y: 1)$ - BREAK for $u / k$.

[^40]$\mathbf{h ( 5 ) : ~ T r a i l i n g ~ i n ~ t h e ~ g a m e , ~} r / y$ sends $r$ to $h(6)$ as T1-B1 [ignoring $h(5)$ ] gets position, and starts a Mini-Gamble. $k$ shot at $y$ and cleared. $y$ went back to position. $u / k$ decided to try to clear this time with $u$ to have played both balls in the period of $h(5)$. $u$ missed, $y$ scored at $h(5)$. (u/k:3, r/y:2) - GAMBLE for $r / y$.
$\mathbf{h}(6): \mathrm{u} / \mathrm{k}$ play first. k goes to position, then y goes to position, joing r who is already there. r can score on its next turn so $\mathrm{u} / \mathrm{k}$ keep u out and has $k$ play a ( 2 -on-1). $k$ clears $r$, and $r$ comes back. $r / y$ can now score with either ball. I thought $u$ would shoot to position at $h(7)$ but instead $u$ tries to clear $r$ and fails. $r$ scores - (u/k:3, r/y:3) - BREAK for r/y. This break was set-up by the Gamble at $h(5)$.
$h(7): u$ shoots to position. y promotes $r$ from $h(6)$ to hoop-shot position at $h(7) . k$ shoots to $h(7)$. $r$ fails the hoop shot, $y$ shoots to clear $k$ but misses, $k$ scores. (u/k:4, r/y:3) - NORMAL for $u / k$.
$h(8)$ : $r$ shoots a little long to $h(8)$. $k$ in clearing position. $y$ shoots to position. $u / k$ could clear with $k$ but choose to bring $u$ into the battle. However u misses, and y scores. (u/k:4, r/y:4) - BREAK for r/y.
$h(9)$ : $k$ shoots to position, $r$ misses, $u$ shoots to position, $r / y$ give up and instead have $y$ shoot to $h(10)$ where it is slightly out of position. $k$ scores and in doing so goes north of $h(10)$. ( $u / k: 5, r / y: 4$ ) - NORMAL for $u / k$.
$h(\mathbf{1 0}): ~ r$ shoots to position, joining $y$ at $h(10)$. shoots to $h(10), y$ goes to position, $k$ shoots and hits $y, r$ scores. (u/k:5,r/y:5) NORMAL for $\mathrm{r} / \mathrm{y}$.
$\mathbf{h ( 1 1 ) : ~ u ~ p l a y s ~ f i r s t ~ b u t ~ d o e s ~ n o t ~ g e t ~ p o s i t i o n . ~ y ~ s h o o t s ~ o v e r ~ t o ~} h(11)$ ready to clear $k$ if it shoots to position. But $k$ also fails to get position allowing $r$ to come over. A (2-on-2) ensues with y ultimately scoring (u/k:5, r/y:6) - BREAK for r/y. r/y win the game. Again the lesson is when you are T1 to a hoop it is vital to get at least one ball in position!

## The SHIP List for AC-GC

AC/GC has a few complicating factors which are reflected by the questions within each of the usual priorities of the SHIP List.

1. Shoot the Hoop: Do I have a ball in position? Have I satisfied the Both Balls Rule or will I with a hoop shot?
2. Hit a Ball: Does Oppo have a ball in position? Have they satisfied the Both Balls Rule or will they with a hoop shot?
a. If I have not satisfied the Both Balls Rule, should I risk playing the second (likely farther) ball?
b. Can I hit Oppo well enough to make a return to position difficult?
c. If Partner-Ball is in position and if playing this ball is required to meet the Both Balls Rule, can I clear the closer OppoBall to protect Partner?
d. If Oppo is not in good position, can I hit (or double-hit) Partner to promote both balls into position?
3. Interfere with the Oppo-Ball:
a. If Oppo's Best Ball has a difficult hoop shot, can I get blocking position (jaws) in front of the Current Hoop?
b. If Partner-Ball is in position and playing Striker is required for the Both Balls Rule, can I block the closer Oppo-Ball to protect Partner?
c. If Partner-Ball was pre-sent to position, I should try to avoid interfering with its hoop shot.
d. Can I put the Striker-Ball in position at the hoop where the closer Oppo-Ball cannot hit it?
4. Take Position at the Hoop:
a. Does the Striker need to be a threat? Should I go close to the hoop or to critical (50/50) distance?
b. Can I get closer than Partner to improve on our Best Ball? Which ball satisfies the Both Balls Rule?
c. Is it time to Gamble at the Next-Hoop or two hoops ahead?
d. If Oppo pre-sent a ball, can I get corner position on it for an effective stop shot next turn?

## Appendix 1

## THE RANDOM-DRAW CHALLENGE



One way to enhance the overall experience of GC and these new games is to play a match consisting of best-of-5 (or best-of-3) where games are selected randomly, by drawing from a bag containing 5 blocks, one for each game. The first game is selected just before the match starts; subsequent games are selected when one game is completed ${ }^{56}$.

[^41]
## Appendix 2

HANDICAPPING WITH THE QUARTER SYSTEM


## Handicapping with the Quarter System

When there is a desire to handicap traditional GC or these new games, we would like to propose that each player receive an allowance of "Quarters" where the quantity he gets is calibrated to his skill level, as measured by handicap/rating, etc. "Spending" a Quarter at a hoop in any of these games would allow the player to contain his shots to only the quarter of the lawn assigned to the hoop in play. In all games, Quarters are best used gross rather than net. This facilitates play across skill levels.

## Use in GC

All players wanting to spend a Quarter at a current hoop must announce their intention to do so before play to it begins. Once spent, a Quarter grants the single player two separate rights:

1. To Shorten their Initial Shot: The initial shot to the Current-Hoop is played as normal or from anywhere along the Half-Way-Line between it and the previous hoop. The Half-Way-Line for $h(1)$ to start the game is between $h(4)$ and $h(5)$; the others are standard. Figure 1 identifies them using white hash marks.

## AND

2. To Shorten Come-back Shots: Figure 2 shows the quarters of the lawn assigned to the nine corner hoops as used in GC. These are delineated by the Half-Way-Lines that run through the peg. Figure 3 shows the quarters of the lawn assigned to four middle hoops. They are delineated by east/west center line together with lines along the Magenta hash marks shown in Figure 1.

If the ball of a player moves outside the relevant quarter for any reason, by his own action or by another player, then he can reverse the path of the ball, returning it to the nearest point on the edge of the quarter. If the ball started outside the quarter and never intersected it, then it can be moved along a line from where it ends up to the current hoop, stopping where it first touches the boundary of the relevant quarter.

At the cost of a single Quarter, a ball can be returned any number of times during the play of a hoop. The return must occur before the next ball plays and this decision stands until the player's next turn, or until his ball is moved again by the play of another ball.


## Benefits of the Quarter System

(1) It is easy to understand and quick to implement.
(2) It handicaps play without providing opportunities to game the system.
(3) It does not require averaging of handicaps for team-play.
(4) In addition to an allowance of quarters, a player can be allocated a single $1 / 2$ Quarter - a "Bit". This would allow gradation of handicaps between Quarters. Spending his Bit allows a player to shorten the initial shot to a hoop; or to shorten come-back shots at that hoop, but not both.

## Data for the Quarter System

Opening shot length, measured in yards, from Half-Way-Lines to centered position on hoops:

| 3.5: | $h(5), h(7), h(11)$ | 10.5: |
| :--- | :--- | :--- |
| 7.0: | $h(3), h(6), h(2), h(4), h(8), h(10)$ |  |
|  | $h(12)$ | 11.5: |
| $h(13)$ [to 1-yard north of $h(3)]$ |  |  |

Straight-on opening hoop shots@
Angled opening hoop shots@

$$
h(2), h(4), h(8), h(10),(10.5 \text { yards })
$$

$h(5)$ and $h(11)$ [11.1 yards, Tangent $=1 / 3$, angle $=18.5^{\circ}$ ]
$h(6), h(12)$ [7 yards)]
$h(7)$ [7.8 yards, tangent $=1 / 2$, angle $\left.=26.6^{\circ}\right]$

## Notes on using Quarters in GC

The best use of Quarters is contextual, psychological, and dependent upon the remaining allowances of Quarters held by all players. That said, here are some things to think about:

1. Going $1^{\text {st }}$ to a hoop is an advantage that may or may not be magnified by spending a Quarter.
2. When playing $2^{\text {nd }}$ to a hoop, you cannot wait to see what your opponent does before making your decision to spend a Quarter.
3. When playing on a particular quarter of the lawn, the farthest you can be from a hoop is 12.6 yards.
4. Distance-wise, you get the most "bang per Quarter" starting on the Half-Way-Line going towards hoops $1,2,4,8,10, \& 13$. These lifts cover 10+ yards from your normal starting point.
5. Quarters are most powerful for offense at $h(6)$ and $h(12)$, and for defense at $h(5), h(7), h(11)$.
6. Unused Quarters do not help win games. Therefore, saving Quarters for the end may not be a good strategy.
7. With three or more Quarters left in your allowance, using them consecutively at $h(5), h(6)$, and $h(7)$ can be very powerful for catching up if you are behind, or pressuring your opponent if you are in the lead.
8. If you are likely to get there, then saving Quarters for $h(11), h(12)$, and $h(13)$ can significantly improve your chances of winning.

## Use in Other Games

With some minor adjustments, the Quarter System just described can be used to handicap all of the games discussed in this book.

1. 2-Shot:The Half-Way-Line applies only to the $1^{\text {st }}$ shot of the $1^{\text {st }}$ turn to a hoop. The right to return a ball to the edge of the current quarter applies after this $1^{\text {st }}$ shot and to all subsequent $1^{\text {st }}$ and $2^{\text {nd }}$ shots.

The $13^{\text {th }}$ hoop is $h(1)$ and not $h(3)$ in 2-Shot. The Half-Way-Line is the north/south line between $h(6)$ and $h(7)$.
2. Dueling Duos: The order of the hoops is random in Dueling Duos. Instead of starting at unfamiliar Half-Way-Lines, a Player spending a Quarter can play his initial shot from where it is relative to the last hoop, or from either penalty spot. The right to return applies to all shots thereafter.
3. 3-Shot: A player can have multiple turns to the Current-Hoop each consisting of a $1^{\text {st }}, 2^{\text {nd }}$, and possibly a $3^{\text {rd }}$ shot. The Half-Way-Line applies before each $1^{\text {st }}$ shot. The right to return applies after each $1^{\text {st }}, 2^{\text {nd }}$, or $3^{\text {rd }}$ shots.
4. AC-GC: No changes are needed.


[^0]:    ${ }^{1}$ Word Version (21-12-09). We chose "he" as our universal pronoun. Please read in whatever works best for you. We understand that errors may be discovered. Please send suggestions to Howard@Sosin.net.
    ${ }^{2}$ We want to thank the Croquet Foundation of America for supporting the many iterations of Croquet Innovations that were played and filmed at the NCC.

[^1]:    ${ }^{3}$ We would like to thank Paddy Chapman for his thoughtful comments and insights.

[^2]:    ${ }^{4}$ David Maloof and Jeff Soo played a series of exhibition matches at clubs in north Carolina in June 2021 highlighting the four new games. The required winning scores have been tweaked based on this experience, mostly in an effort to control the length of games.

[^3]:    ${ }^{5}$ We borrowed this phrase from the great sports announcer, Warner Wolf.
    ${ }^{6}$ Many thanks to Russ Dilley for allowing us to copy CIT-related videos from his site to ours.
    ${ }^{7}$ I want to thank the participants in CIT II (March 2019) for input on 2-Shot and for playing it while the rules were still being worked out. They are - David Bent, Jamie Burch, Paddy Chapman, Danny Johnston, Stephen Mulliner, Ben Rothman, and Pete Trimmer.
    ${ }^{8}$ Time only permitted two games during the initial videoing. This suggested that tweaking the scoring system for DD would be desirable. We changed it from 11 win by 2 , used in the original two videos, to the first team to 8 wins for the remaining two videos.

[^4]:    ${ }^{9}$ Here the "Spent" Ball is the ball that played just before Striker and the "Danger" Ball is the ball that will play just after Striker.

[^5]:    ${ }^{10}$ Additional useful/cocktail information: The outer dimensions of the lawn, measured in units of 7 yards, is $4 \times 5$ and the inner rectangle formed by the hoops themselves, the "box", is $2 \times 3$

[^6]:    ${ }^{11}$ Two things are not discussed but are important for future study: (i) The distributions associated with CDs - weaker players have lower CD's and higher variances; and (ii) Psychology - for whatever reason there will be days when longer shots are (seem) easier than short ones ...

[^7]:    ${ }^{12} h(1)$ relative to $h(12)$ matters when there is wrap-around from $h(12)$ back to $h(1)$.

[^8]:    ${ }^{13}$ Stephen Mulliner coined the name Stepping-Stone. Another way to think of it is as a "Transportation" Ball.

[^9]:    ${ }^{14}$ We want to thank Jeff Soo for the suggestion that 2-Shot use the opening that Reg Bamford suggested for GC.
    ${ }^{15}$ We suspect that a lot of 2-Shot will be played starting at $h(1)$, which simplifies without compromising the game.

[^10]:    ${ }^{16} \mathrm{k}$ does not have to stay in bounds with this croquet shot in order to earn a $2^{\text {nd }}$ shot.

[^11]:    ${ }^{17}$ Jeff Soo thought of this opening.

[^12]:    ${ }^{18} r$ has other good plays. For example, $r$ could promote $y$ and maybe leave himself a hoop shot or the possibility of becoming y's Pioneer.

[^13]:    ${ }^{19}$ This match was played when the required winning score was set at 7 points. We have since changed/reduced it to just 5 , finding that this change maintains the drama of a game without dragging out its length.

[^14]:    ${ }^{20}$ Perhaps surprisingly for CDs of 9 and 12 , but not 15 , moving the placement of the Stepping-Stone can improve the results, but not enough to overturn the dominance of shooting. For example, put the Stepping-Stone ( $r$ ) 4 yards ahead of y leaving 16 yards to the target ( $u$ ) -- the same result is obtained if the Stepping-Stone is at 16 yards leaving 4 yards to the target - Here the calculations are: [CD=9: 25.8\%], [CD=12: 37.0\%] and CD=15: 46.8\%].

    Clearly the slope of the $\mathrm{CD} /$ distance curves matter. As a final calculation, at $\mathrm{CD}=12$ the probability of hitting from a 10 yard Stepping-Stone would need to increase from $58.2 \%$ to $72.7 \%$ to match the result from shooting.

[^15]:    ${ }^{21}$ It is certainly the case that immediately after scoring a ball is on the non-playing side of a hoop and is (probably) momentarily wired from balls in front of the hoop on the playing side. This weakness is mitigated by the ability of a just-scored ball to shoot back through the hoop.
    ${ }^{22}$ Another way to Approach These Questions: Consider a "new" game: One that follows GC rules - all balls play to each hoop in normal continuing rotation, but only one ball of a team can score at the Current-Hoop and, before play to a new Current-Hoop begins, each team flips a coin to determine which one of its two balls will be its "scoring-ball". This game is identical to what occurs in Dueling Duos once a ball of each team has scored. That is, in the new game, asking if k (the designated "scorer" for $\mathrm{u} / \mathrm{k}$ ) would rather face r or y at the Current-Hoop, is the same as assuming in Dueling Duos that u has scored the Current Hoop and asking if it is better for $u / k$ (and worse for $r / y$ ) if $r$ scores first or if $y$ scores first for $r / y$.

[^16]:    ${ }^{23}$ In hockey a team that is losing can remove their goalie to add offense. We have found no equivalent in DD.

[^17]:    ${ }^{24}$ The averages were computed after removing the turn with the longest time from each category. In my mind, for various reasons they were not representative. In fairness to the $1^{\text {st }}$ turn, I deducted 30 seconds from the times of the $1^{\text {st }}$ scores to account for the time consumed in selecting a new hoop.

[^18]:    ${ }^{25}$ Note that in this game the $1^{\text {st }}$ and $2^{\text {nd }}$ hoops were played at $h(12)$ and $h(5)$, the same physical hoop but in different directions.

[^19]:    ${ }^{26}$ Sherif Abdelwahab deserves credit for dissuading us from a more complicated earlier version of this game where not only did offsides exist, but it was assessed after every turn, even if there was not a score!

[^20]:    ${ }^{27}$ Offensive Strategies involving fewer than 3 shots are available and are discussed at the end of this section.
    ${ }^{28}$ There are never more than 3 shots in a turn. Scoring a hoop does not generate another shot. However, scoring a hoop does not prevent completing all three shots of a turn if the hoop is scored with the first or second shots.

[^21]:    ${ }^{29}$ Note that if y is in the jaws at the start of y 's turn (after a series of misses by opponents and partner!), y must roquet a ball before scoring.

[^22]:    ${ }^{30} u$ would need to roquet $y$ or $r$ and then scatter the balls leaving $r$ separated from both $y$ and $u$.

[^23]:    ${ }^{31}$ Even though $u$ and $y$ are in contact, Striker does not have free placement of $u$ around $y-i$ must be on the yard-line. In order for $u$ to earn a $3^{\text {rd }}$ shot, its $2^{\text {nd }}$ shot must cause y to move, so u must hit into y a bit.
    ${ }^{32}$ We have ignored the possibility that $u$ does not roquet $y$. If $u$ did miss $y$, or choose not to shoot at $y$ after the $1^{\text {st }}$ shot, then, assuming $u$ does not hit another ball, u's turn would end after his $2^{\text {nd }}$ shot.

[^24]:    ${ }^{33}$ I want to thank Stephen Mulliner. His concern for clarity led to the term "Period of a Hoop".

[^25]:    ${ }^{34}$ There is nothing inherently special about $u / k$ going first. However, one team or the other had to be chosen to have the Winning Rule make sense.

[^26]:    ${ }^{35} \mathrm{u}$ or k could clear $r$, but that would be of little advantage unless $u$ does it and goes to position at the same time.
    ${ }^{36}$ A team identified with a " 2 " has satisfied the BBR, the "*" is implicit. Thus, a (2-on-2) is equivalent to (2*-on- $2^{*}$ ).

[^27]:    ${ }^{37} \mathrm{k}$ could play. But the best k could do is to shoot to position.
    ${ }^{38}$ This possibility does not exist in GC. It arises in AC-GC because balls are marked in one yard and are always in play.

[^28]:    ${ }^{39}$ The (2-on-1) has its own derivative - (1*-on-1). Here one ball has met the BBR but is far from the Current-Hoop and stays out of play, at least initially.

[^29]:    ${ }^{40} y$ can win at an even-numbered Current-Hoop with a long scoring shot as his $1^{\text {st }}$ shot in the Period of the Current-Hoop.
    ${ }^{41}$ Sometimes the decision will be made for you. For instance, if in scoring the last hoop your ball does not have a good shot to the next hoop.

[^30]:    ${ }^{42} r$ could try for the best of both worlds - shooting gently at $u$, attempting to knock it out of position, while staying close if the roquet succeeds or fails.

[^31]:    ${ }^{43}$ Alternatively, $r$ can stay put and $y$ can enter. Probably best is for $y$ to shoot to a position that is north and slightly east of $h(1)$ from which he can drive $u$ away from his position in front of $h(1)$ or out of the jaws. If $u$ still chooses to jaws, then $y$ will clear $u$, followed by $k$ entering starting a (2-on-2).

[^32]:    ${ }^{44}$ It is important to remember that even when it is action by $r / y$ that scores the hoop for $u / k, r / y$ still goes first in the next Period.

    45 There are two things to note. (i): $r$ needs to be south of $h(1)$ in order to attempt this peel. Presuming $u$ shot first and found position, this means that if $r$ wants to shoot at $u$ to enter the game, then $r$ should do so from the northern-most spot on the Start-in Line. If $r$ fears $u$ jawsing, then cuddling is better than shooting. (ii): if $r$ fails on the peel, then $u$ can make $h(1)$ and proceed to $h(2)$ setting up a fluid and complicated situation for both teams that could go either way, depending on whether $y$ or $u$ are in position and what $r$ and $k$ can accomplish.
    ${ }^{46}$ Clearing y in the same stroke would be both glorious and unlikely, but worth a try ...

[^33]:    ${ }^{47}$ We did not differentiate between even/odd hoops, or long/short hoops but give "generic" results for CH and NH hoops. Additionally, we did not factor in Jawsing or the ability to have a ball clear and get position at the same time.

[^34]:    ${ }^{48}$ The calculations are made easier by assuming that winning a (2-on-1) to get to a (2-on-2) occurred with the same fraction as getting there directly by clearing, and that both had a value of $1 / 3$. This meant that we only had to evaluate one of these options when both were possible.

[^35]:    ${ }^{49}$ Dealing with the product of probabilities leads to some counter-intuitive results. For example, suppose the $\operatorname{Prob}(A)=.8$ and the $\operatorname{Prob}(B)=.2$. Here the sum of the $\operatorname{Prob}(A)+\operatorname{Prob}(B)=1.0$ and the $\operatorname{Prob}(A)^{*} \operatorname{Prob}(B)=.16$. Now suppose $\operatorname{Prob}(A)=.6$ and $\operatorname{Prob}(B)=.4$. $\operatorname{Then} \operatorname{Prob}(A)^{*} \operatorname{Prob}(B)=.24$ even though sum of the probabilities remains the same. What matters is the percentage change in probabilities. If $\operatorname{Prob}(A)=.4$ and $\operatorname{Prob}(B)=.4$ then $\operatorname{Prob}(A) * \operatorname{Prob}(B)=.16$. Here one halved while the other one doubled leaving the product the same.

[^36]:    ${ }^{50}$ The ability to tack-on an additional Gamble (or Gambles) adds value to the original Gamble that is not factored into our calculations but has practical value.

[^37]:    ${ }^{51}$ If $k$ ignores an in-position $r$ and goes to position, then $y$ will play. If $y$ finds position, then $r / y$ will have two balls in position at $h(3)$ and should win the hoop! If $k$ clears $r$ but does not get position, then $r / y$ will follow with T2-B1 by attempting to send their closest ball to position at $h(3)$. All eyes will turn to $u$. As T1-B2, $u$ will shoot for position at $h(3)$. If he succeeds, then there will be a (2-on-1) at $h(3)$, with $u / k$ as the 2 -ball team. If $u$ fails to get to position, then $y$ will lag to $h(3)$ and start a (2-on-2).
    ${ }^{52}$ Earlier we considered the situation where $u / k$ sends both of its balls to $h(1)$ and gets only one of them, $u$, to position. $r / y$ have $r$ at $h(1)$ and it is $y$ to play. We showed that $r / y$ would be indifferent between having $y$ attempt to clear (with a $33.3 \%$ success rate) or engage in a (2-on-1) as the 1-ball team, (again with a $33.3 \%$ success rate) as both of these get $r / y$ to a ( $2-$ on- 2 ). But here $r / y$ are in a more advantageous position. It has a ( $1^{*}-o n-1$ ) that should be played out. Our estimate is that $r / y$ should win $45 \%$ of the time, instead of $33.3 \%$, because $k$ is off at $h(2)$.

[^38]:    ${ }^{53}$ This assumes that k did not have position at $\mathrm{h}(2)$, Figure AC-95.

[^39]:    ${ }^{54}$ You can still be thwarted if Oppos get both balls to position!

[^40]:    ${ }^{55}$ Sadly, I was wrong when I identified r's giving up at $h(2)$ and going to $h(3)$ as a Gamble. Once again it was not. Additionally, my commentary was incorrect and confusing for much of the play at this hoop in part because I thought $u$ had scored when he had not.

[^41]:    ${ }^{56}$ One way to simplify this challenge would be to harmonize the rules of GC and AC with a game that could be called " 1 -shot". It starts with GC rules but modifies them to have AC rules for mark-in balls and for striking faults.

